Due Monday, Jan. 26

0. Read Chapter 1. Skim Part I on Tools (Chapters 2-4).

1. Sketch the wave function $\psi(x)$ and the probability distribution for the $n = 1$ state of the infinite one-dimensional square well. (a) Qualitatively sketch how the wave function would change if the well were not infinite but still large? (b) How would the energy of a given state of the infinite well change if the well were finite? (c) How does the ground state energy and wavefunction change if the particle’s mass is doubled?

2. A particle of mass $m$ is in the ground state of an infinite one-dimensional potential well of length $D$ centered at the origin (the walls are at $\pm D/2$). What is the probability that the particle is in a small interval $\delta x = 0.001D$ at location (a) $x = 0$, (b) $x = D/4$, and (c) $x = D/2$? (Since $\delta x$ is so small, no integrals are needed to get the answer.)

3. A virus of mass of $10^{-7}$ grams is moving with a speed of 1 mm/s in a confined region 0.001 mm long. If we were to treat this system as an infinite one-dimensional well, estimate the value of the quantum number, $n$.

4. Calculate the expectation value of $<p>$ and $<p^2>$ for the ground state of a particle of mass $m$ confined by an infinite square well of width $D$.

5. Start with a particle of mass $m$ in an infinite square well centered at the origin as in problem 2. A weak perturbing potential of the form $V(x) = C_1x$ is now introduced. (a) Find the first-order energy shift of the ground state. (b) Answer the same question if the perturbing potential is of the form $V(x) = C_2x^2$.

6. Suppose a particle of mass $m$ is in an infinite square-well potential as in problem 2 and is initially in the ground state when the walls of the well are suddenly moved apart so that the well width becomes $2D$. What is the probability the particle is in the ground state of the new well? (Hint: The wavefunction of a particle cannot instantly change.)

7. Two distinct particles of equal mass are placed in the same one-dimensional infinite square well. (a) Without re-solving the Schrödinger equation, write down the energy eigenvalues and eigenfunctions for this system. (b) What is the first-order shift in the total ground state energy due to a harmonic interaction between the two particles given by $V(x_1, x_2) = \frac{1}{2} k (x_1 - x_2)^2$?

8. Consider the potential shown in the figure of problem 5.38. This potential approximates that seen by an alpha particle confined to a heavy nucleus like radium. It is attractive inside the nucleus, but once outside, the alpha particle experiences the coulombic repulsion. (a) Sketch the potential and the ground and first excited state wavefunctions and indicate the regions in energy where stable bound states exist. (b) Sketch the wavefunction for a particle with an energy slightly greater than $V_\infty$ and explain what will happen to a particle of this energy if initially confined to the well?