Math 484: Homework 2

Due: 9 February, 2011

All equation references are from the notes given in class.

1.
   a. Verify (19).
   b. Show that the roots of the quadratic equation (20) are real.
   c. Show that both roots are negative if and only if both b and c are positive.
   d. Show that since f, D, and χ are positive, if $c > 0$ then $b > 0$. Thereby deduce (21).

2. **Investigate the changes in the analysis that would occur under the following conditions.**
   a. If μ and χ were functions of ρ.
   b. If k and f were functions of ρ.

3. **If two-dimensional variation is considered,** the governing equations (10) and (11) are replaced by

$$\frac{\partial a}{\partial t} = \nabla \cdot \left(\mu \nabla a - \chi a \nabla \rho\right), \quad \frac{\partial \rho}{\partial t} = fa - k\rho + D\nabla^2 \rho.$$

   Assume disturbances with spatial dependence $\sin(q_1 x + q_2 y + \theta)$, where q_1, q_2, and θ are constants. Show that if $q^2 = q_1^2 + q_2^2$, then $2\pi / q$ remains the disturbance period and the instability condition remains (22).

4. Define Δ by $\chi a_0 f / \mu k = 1 + \Delta$ and suppose that Δ is small and positive. [Compare (22).] Ignoring higher order terms in Δ, find an approximation for the larger root of the quadratic. Deduce that the wavelength of the fastest growing disturbance is approximately $2\pi(2D / k\Delta)^{1/2}$.