MATH225, Fall 2009 **Name:**
Worksheet 6 (3.1, 3.2, 3.3) **Section:**

For full credit, you must show all work and box answers.

1. Given

$$\frac{dx}{dt} = 2x + 7y$$
$$\frac{dy}{dt} = -x - 6y$$

(a) Is this system linear?

(b) Rewrite the system in matrix-vector form.

(c) Show $\mathbf{Y}_1(t) = \begin{pmatrix} e^{-5t} \\ -e^{-5t} \end{pmatrix}$ and $\mathbf{Y}_2(t) = \begin{pmatrix} -7e^{t} \\ e^{t} \end{pmatrix}$ are solutions to the system.

(d) Show $\mathbf{Y}_3(t) = \begin{pmatrix} e^{2t} \\ e^{2t} \end{pmatrix}$ is not a solution to this system.

(e) Show $\mathbf{Y}_1(t)$ and $\mathbf{Y}_2(t)$ are linearly independent.

(f) Find the general solution to the system. What principle are you using to do this?

2. Given the system

$$\begin{aligned}\frac{dx}{dt} &= 2x + 7y \\ \frac{dy}{dt} &= -x - 6y\end{aligned}$$

(a) Is this system linear?

(b) Find the general solution.

(c) Sketch the phase portrait. Make sure you include the straight-line solutions and at least two other trajectories. Classify the origin.

(d) Find the particular solution that satisfies the initial condition $(x(0), y(0)) = (6, 0)$. Report your solution as one real vector.

3. Given the system

$$\mathbf{Y}' = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \mathbf{Y}$$

(a) Find the general solution.

(b) Sketch the phase portrait. Make sure you include the straight-line solutions and at least two other trajectories. Classify the origin.

(c) Find the particular solution that satisfies the initial condition $\mathbf{Y}(0) = (1, 5)$. Report your solution as one real vector.

4. Given the system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & 0 \\ -4 & 5 \end{pmatrix} \mathbf{Y}$$

(a) Find the general solution.

(b) Sketch the phase portrait. Make sure you include the straight-line solutions and at least two other trajectories. Classify the origin.

(c) Find the particular solution that satisfies the initial condition $\mathbf{Y}(0) = (-3, 1)$. Report your solution as one real vector.