MATH225, Fall 2011 **Name:**
Worksheet 9 (7.3, 7.4, 7.5) **Section:**

For full credit, you must show all work and box answers.

1. Find the Laplace transform of the following functions. (Use the table.)

   (a) $g(t) = e^{2t}\cos(3t)$

   (b) $g(t) = 4t^2e^t$

   (c) $h(t) = te^{6t}\sin(3t)$
   Use Formula 10

   (d) $f(t) = 7e^tU(t-2)$

   (e) $f(t) = t^2U(t-3)$

   (f) $h(t) = \begin{cases} e^{2t}, & 0 \le t < 1 \\ 6t, & t \ge 1 \end{cases}$
   Hint: Write $h(t)$ in terms of Heaviside functions first.

2. Find the inverse Laplace transform of the following.

(a) $F(s) = \dfrac{s+4}{s^2+2s+10}$

(b) $G(s) = \dfrac{e^{-3s}+1}{(s+4)(s+1)}$

(c) $G(s) = \dfrac{e^{-2s}}{s^2-4s+9}$

3. Find the Laplace transform of the following functions. (Use the table.)

   (a) $h(t) = e^{2t} * \cosh(3t)$
   Do not evaluate the integral before transforming.

   (b) $h(t) = \int_0^t \tau \sin(\tau) d\tau$
   Do not evaluate the integral before transforming.

4. Find the inverse Laplace transform of $H(s) = \dfrac{4}{(s+2)(s+1)}$ using:

   (a) Partial Fractions

   (b) The Convolution Integral. Simplify your solution. Your final answer should not contain integrals or convolutions.

5. Given $y'' + 9y = \cos(3t), \quad y(0) = y'(0) = 0$. Solve using

(a) The Method of Undetermined Coefficients.

(b) Laplace Transforms.

(c) Assuming this initial-value problem models a spring-mass system, what phenomenon is occurring?

6. Use the Laplace transform to solve the following equation. Simplify your solution. Your final answer should not contain integrals or convolutions.
$y'(t) + \int_0^t y(t-\tau)d\tau = t, \quad y(0) = 0$

7. Solve the following initial-value problem. Simplify your solution. Your final answer should not contain integrals or convolutions.
$y' + 4y = \delta(t-\pi) + \delta(t-2\pi), \quad y(0) = 1.$

8. Consider the LRC series circuit with $R = 40\Omega$, $L = 1$ h, $C = 0.001$ f. A battery supplying 90 V is attached to the circuit. The switch to the battery is initially closed and left closed for 1 second. At time $t = 1$ it is left open. Initially, $q(0) = i(0) = 0$. The differential equation that describes this situation is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t), \quad E(t) = \begin{cases} 90, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$$

Solve the initial-value problem. Simplify your solution. Your final answer should not contain integrals or convolutions.