Extension of the Rotated Elastic Parabolic Equation to Beach and Island Propagation

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***Abstract*—Improvements in the capability of handling sloping interfaces and boundaries with the parabolic equation method have been an active area of research. Recent progress in accurately treating range-dependent seismoacoustic problems has involved coordinate transformation techniques. The variable-rotated parabolic equation is among recent advances in this area. The solution rotates the coordinate axes to achieve greater accuracy in the presence of range-dependent bathymetry. At points of slope change the rotated solution interpolates and extrapolates the field into adjacent regions. This approach is extended to solve problems involving variable topography (above-ocean-surface sediments) by accounting for the transition and boundary conditions at the water/solid/air interfaces. It is applied to range-dependent problems of sound transmission up a beach and through an island. The method is benchmarked for accuracy against a finite-element solution.**

***Index Terms*—Parabolic equation method, range dependence, sea-to-land propagation, seismoacoustics.**

I. Introduction

THE numerical treatment of range-dependent fluid-solid interfaces is one of the prominent issues in underwater acoustics. Realistic ocean bottoms are often characterized by frequent changes in bottom slope. In some cases, such as deep water, it is suitable to approximate the bottom as horizontal. In shallow-water environments, the effects of bottom features are often compounded because the sizes of the features relative to length scales of the waveguide are large. Many acoustics problems in shallow water feature bathymetry that varies with range; therefore, the ability to handle multiple sloping interfaces in a single problem is a necessary capability to accurately calculate the acoustic field. A natural boundary to shallow-water sound propagation is a beach environment at which sound can be transmitted from either onshore or offshore into the adjacent region. The ability to model this type of environment could be used in earthquake/blast monitoring or tracking military shore movements. An additional motivation for modeling such an acoustic environment, in an underwater scenario, would be the detection of targets hidden behind islands, seamounts, or atolls.

The parabolic equation method is an efficient solution technique for problems with outward energy propagation. A parabolic wave equation is derived by factoring the elliptic Helmholtz equation into a product of incoming and outgoing waves, and assuming that outgoing energy dominates. This assumption is that two-way propagation effects are not significant and is valid for typical bottom slopes encountered in the ocean environment ($<5^\circ$). The assumption will break down as backscattered energy begins to significantly contribute to the total field.

A historically effective treatment of a sloping interface is to approximate in terms of a series of range-independent stair steps [1], though more accurate mechanisms for treating range dependence are coordinate transformation techniques [2]–[6]. Progress has been made recently with coordinate transformation techniques for treating variable sloping bathymetry accurately. Two types of techniques have been used successfully: rotated [2] and mapped coordinates [3]. When introduced in 1990, the rotated solution became the first parabolic equation solution capable of handling a sloping fluid–solid interface to high accuracy. The solution involves rotating the coordinate axes so that they are aligned with the slope of the ocean bottom. The rotation eliminates problems associated with stair steps by replacing the sloping bottom interface with a sloping surface that is simpler to handle, since the pressure release condition at the surface is more easily applied than are range-dependent bottom interface conditions [2]. The rotated solution has recently been generalized to treat variably sloping bathymetry [7]. At points of slope change, the variable-rotated solution employs an interpolation and extrapolation scheme to advance solutions past points of slope change. This approach has been applied to extend both fluid and elastic rotated parabolic equations to variable slopes.

The rotated solution was initially restricted to bottoms of constant slope, so other techniques for handling range dependence were developed. The mapped solution is capable of treating variable bathymetry, and has provided accurate solutions when range dependence is sufficiently gradual [3]. The solution is based on mapping the ocean bottom to a horizontal line. As with rotated coordinates, mapped coordinates replace a sloping bottom interface with a sloping ocean surface to avoid a stair-step approximation. In contrast to the rotated solution, the mapping gives rise to additional terms in the wave equation that are difficult to handle in a parabolic equation solution. However, the

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effects of these terms can be taken into account for small slopes. For larger slopes, the solution breaks down and can become inaccurate beyond points of slope change [3].

The parabolic equation method was extended in 1995 to handle problems involving a sloping ocean bottom that continues beyond the water surface and onto a beach (topography) [8], though this was only for a fluid bottom using a stair-step approximation. The mapped solution has recently been extended to handle bottom topography that rises above the surface of the ocean [9]. Coupling between ocean acoustic and sediment propagation can be significant if there is a mechanism for trapping energy near the surface, such as ducting by a sediment layer or upward refraction. These effects compound the need for an accurate sediment model capable of treating elastic sediments above the sea surface. The variable-rotated solution is capable of providing a more accurate solution than the mapped solution, particularly in the presence of large slopes.

In this paper, the variable-rotated parabolic equation solution is extended to handle a class of problems involving variable surface topography. The extension of the variable-rotated solution enables the computational grid of the method to continue above the ocean's surface. Free surface boundary conditions are applied in the extended regions. In Section II, current treatments of variable sloping interfaces are discussed, and the rotated solution is benchmarked against a normal mode solution to demonstrate improved accuracy over the mapping approach. Surface topography is discussed in Section III, and the rotated solution is generalized to treat problems involving variable topography. In Section IV, the new approach is benchmarked against a finite-element solution and applied to model environments involving variable slopes in bathymetry and topography.

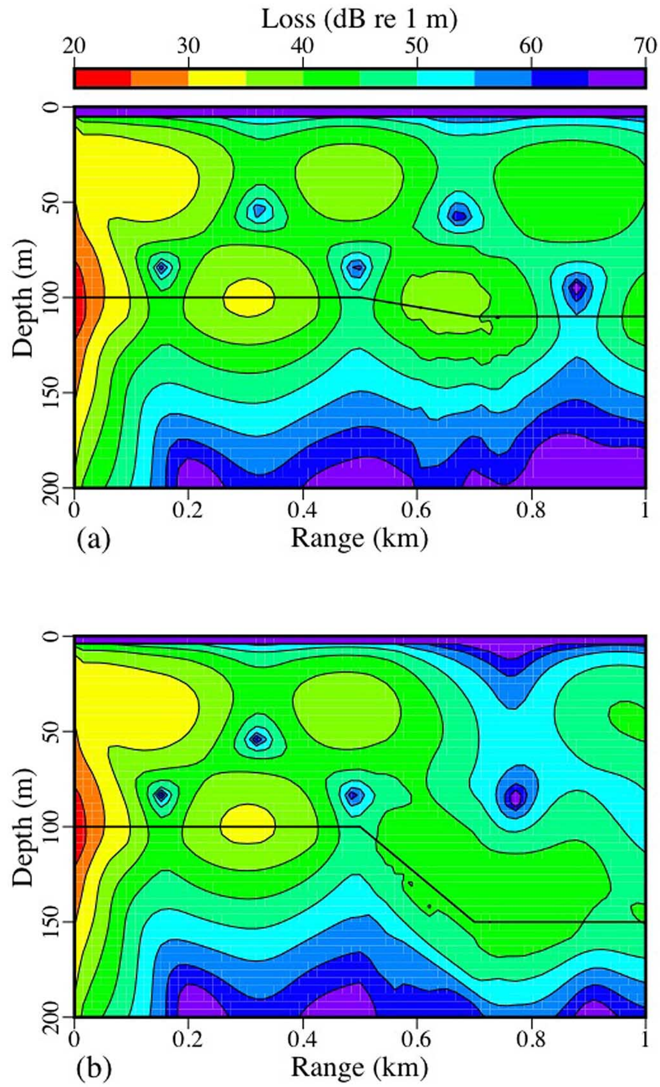

Fig. 1. Transmission loss contours computed using a parabolic equation solution for the compressional field of a 15-Hz source at 95 m, where the downward slope between 500 and 700 m is varied: (a) 2.9° slope and (b) 14.0° slope.

II. Variable Boundary Waveguides

A traditional technique for treating a sloping bottom interface is to use a stair-step approximation to give range-independent regions of solution. This technique introduces artificial vertical interfaces between the regions and it becomes necessary to account for energy transfer across these interfaces. A leading-order energy-conservation correction can be applied at every vertical interface, but errors associated with this correction accumulate over the entire range-dependent interval and can affect the pressure amplitude [10].

More modern techniques for treating sloping interfaces involve coordinate transformations. By applying a change of variables to map the ocean-bottom interface to a horizontal line, mapped coordinates can be obtained. Additional terms that arise in the wave equation as a result of this change of variables are neglected. The combination of the mapping and neglect of slope terms is equivalent physically to distorting the waveguide and can be problematic unless accounted for. However, the solution conserves energy since no stair steps are used. A result of the mapping is that points of slope change, other than at the surface, are not in the new domain. If these points are not considered in the final solution, then major features of the field can be lost. Consequently, a correction is applied to the mapping solution at these points of slope change. For problems involving relatively steep slopes ($>2.5°$), the correction is necessary to account approximately for effects of neglecting the introduced slope terms [3]. The leading-order mapping correction

$$p(r,z) \approx p(r,z)e^{-ik_0 z \sin(\delta)} \tag{1}$$

is applied each time there is a change in slope δ. The form of (1) indicates that this correction affects only the phase of the solution.

The rotated parabolic equation solution also avoids a stair-step approximation, and thus conserves energy, by aligning the coordinate axes to the slope of the ocean bottom. At a transition between regions of constant slope, the incident field at the end of the old region must be translated to obtain the transmitted field at the start of the new region. A local correction could be applied at points of slope change, but this would be analogous to using the mapping correction. Greater accuracy is achieved by interpolating and extrapolating the incident field into the new

TABLE I
ELASTIC PROPERTIES OF THE LAYERS USED IN EXAMPLES A AND B, WHERE THE WATER LAYER IS DENOTED AS THE FIRST LAYER

Example	Layer	c_p (m/s)	c_s (m/s)	ρ (g/cm^3)	α_p (dB/λ)	α_s (dB/λ)
A	1	1500	0	1.0	0.0	0.0
	2	1650	660	1.58	0.045	0.024
	3	1705	684	2.03	0.1	0.04
	4	1800	720	2.25	0.2	0.1
B	1	1500	0	1.0	0.0	0.0
	2	2400	1200	1.5	0.05	0.1
	3	3400	1700	1.8	0.1	0.2

region of constant-slope bottom. At a change in slope, initial conditions are obtained on a line that is orthogonal to the new bottom interface. The parabolic equation is used to march the solution in range through the region. The initial conditions are obtained by interpolating part of the solution from the current region into the new region and the remainder of the initial conditions are obtained by extrapolating the solution a short distance by overshooting the point where the slope changes. The rotated coordinates technique eliminates problems associated with stair steps and does not give rise to extra terms in the wave equation, as does coordinate mapping. Since no correction is applied and the field is naturally propagated past points of slope change, the variable-rotated solution should be more accurate in the presence of multiple sloping fluid–solid interfaces and provide improved accuracy over the mapping approach.

An example illustrates differences between the mapped and variable-rotated solutions and demonstrates that the rotated solution is capable of handling larger slopes than the mapping solution. In order to compare the two solutions against a normal mode solution, the sediment is treated as a fluid, and fluid sediment model implementations of the mapping and rotated solutions are used. The normal mode solution treats the range-dependent bottom interface using a stair-step approximation with a single-scatter correction [11]. The variable-rotated, normal mode, and mapped solutions are compared for the sample problem featuring a downsloping step. The ocean depth is initially 100 m, from 0 to 500 m in range, and then slopes downward from 500 to 700 m in range. The bottom depth at 700 m was varied between 110 and 150 m to illustrate differences. Sample propagation calculations for the compressional field resulting from a 15-Hz source at 95 m are given in Fig. 1 for 2.9° and 14° slopes. The ocean bottom is a fluid halfspace in which density is 5.0 g/cm^3 and compressional sound speed is 5000 m/s. Compressional attenuation is neglected and an artificial attenuating layer prevents spurious reflections from the bottom of the computational domain. Sediment geoacoustic parameter values are chosen large to prevent bottom penetration and encourage acoustic energy reflection into the water so that differences in solution may be more readily observed.

Transmission loss line plot comparisons for the downslope problem, at receiver depth 95 m, are given in Fig. 2, where the depths at 700 m are 110 and 150 m. The generalized rotated parabolic equation solution (solid line) is compared against the coupled mode (dashed line) and the mapping (dotted line) solutions. For the modest slope in Fig. 2(a), the three solutions agree very well. Fig. 2(b) shows comparisons for a very steep slope case, for which the variable-rotated and coupled mode solutions are in agreement. However, effects of the larger slope cause the mapping solution to disagree with the other two. In particular, the mapping solution is out of phase and generally in poor agreement beyond the downslope.

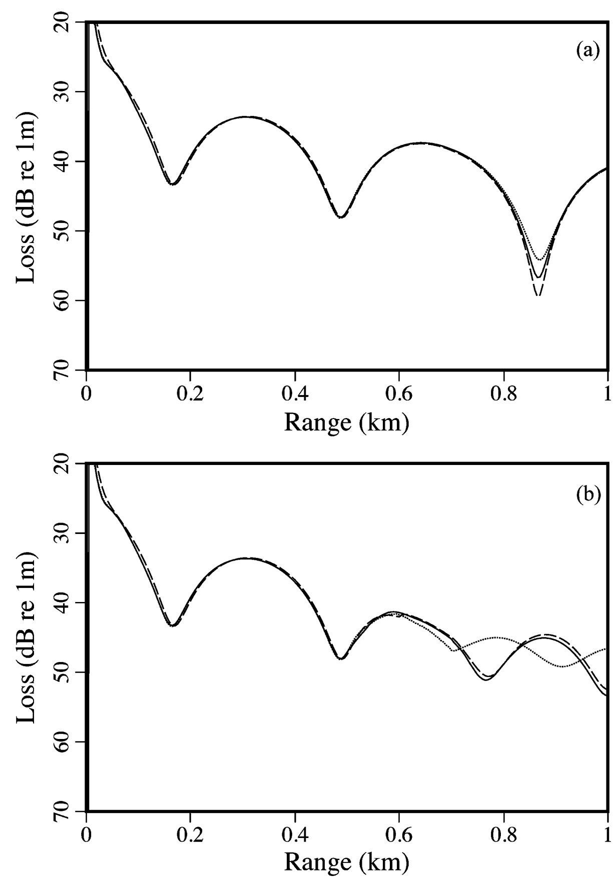

Fig. 2. Transmission loss versus range for the downslope example, at receiver depth 95 m. Comparisons show the variable-rotated solution (solid curve), the coupled mode solution (dashed curve), and the mapped solution (dotted curve), for slope angle: (a) 2.9° and (b) 14.0°.

III. VARIABLE TOPOGRAPHY

The main contribution of this work is to extend the variable-rotated solution technique to problems involving variable

TABLE II
BATHYMETRIC VALUES FOR EXAMPLE A

Range (m)	Bottom Depth (m)
0	63
223	60
334	54
501	47
600	47
1444	30
1600	18
1732	9
1889	0
2111	-6
2150	-7
2200	-7
2300	-10
2400	-14
2500	-18

TABLE III
BATHYMETRIC VALUES FOR EXAMPLE B

Range (m)	Bottom Depth (m)
0	300
1000	320
2000	350
3000	360
4000	340
5000	310
6000	285
7000	220
8000	130
9000	50
10000	0
12000	-40
15000	-80
18000	-40
20000	0
21000	20
22000	50
23000	60
24000	100
25000	140
26000	185
27000	220
28000	280
29000	330
30000	350
32000	380
35000	390

sediment interfaces that extend above the surface of the ocean (topography). The rotated solution is currently capable of treating variable bathymetry, but cannot handle topography. For the latter case, appropriate free surface boundary conditions must be enforced at the new interface. We use a rotated version of the (u_r, w) formulation of elasticity [12], [13], where u is the horizontal (tangential) displacement and w is the vertical (normal) displacement, in a cylindrical (r, z) coordinate system. The elastic parabolic equation has the form [12], [13]

$$\frac{\partial}{\partial r}\begin{pmatrix} u_r \\ w \end{pmatrix} = L\begin{pmatrix} u_r \\ w \end{pmatrix}, \qquad u_r = \frac{\partial u}{\partial r} \tag{2}$$

where the matrix L is a depth operator.

In problems with topography, there are at least three distinct sets of boundary conditions that must be satisfied along the fluid–elastic, air–fluid, and air–elastic interfaces. In addition, problems that have sediment layering have a fourth set of boundary conditions that must be satisfied along elastic–elastic interfaces. With the rotated coordinate approach in the (u_r, w) formulation, the parabolic equation method accurately treats all four sets of interface conditions [13]. When the solution transitions from sub-ocean surface to super, fluid–solid interface conditions are replaced with air–solid (free surface) boundary conditions. At the air–water interface, the pressure–release condition $p = 0$ is applied. At the air–sediment interface, free elastic-surface boundary conditions require that both the normal and tangential stresses vanish. For Lamé parameters λ and μ, descriptive of the medium, these are

$$\lambda_b u_{rb} + (\lambda_b + 2\mu_b)\frac{\partial w_b}{\partial z} = 0 \tag{3}$$

$$\frac{\partial}{\partial z}(\lambda_b u_{rb}) + \frac{\partial}{\partial z}\left((\lambda_b + 2\mu_b)\frac{\partial w_b}{\partial z}\right) + \rho_b \omega^2 w_b = 0 \tag{4}$$

where the subscript b denotes the elastic layer. At the fluid–elastic interface, three interface conditions must hold: continuity of vertical displacement, continuity of normal stress, and zero tangential stress. These conditions can be implemented numerically using Galerkin's method or finite differences as described in [14]. For the case of a fluid sediment model, boundary conditions pertaining to the tangential stress are not present and hence it is only necessary to maintain the conditions on the normal stress and the vertical velocity.

For problems with variable topography, the rotated solution employs the procedure described in [7] to treat a variably sloping interface, the primary difference being the boundary conditions that are enforced at the surface. The variable-rotated solution approximates the air–sediment interface as a sequence of constant slope regions. The field is propagated through a given constant slope region by the parabolic equation in a coordinate system that has been rotated to align with the interface. The interface is treated as range independent, and at a slope change in the topography it is sufficient to either exclusively interpolate or extrapolate the field in the sediment layer, while no operation is necessary in the air above the sediment (where the pressure is zero). As described in [7], the solution is marched past the point of slope change until it can be interpolated and extrapolated onto the new grid, after which the coordinates are rotated and the solution is marched through the first region of constant surface slope. It is expected, based on the more accurate treatment of slope changes and range dependence, that the variable-rotated solution can handle variable topography with greater accuracy than either mapping or energy conservation techniques.

IV. Benchmarks and Applications

In this section, the variable-rotated solution is applied to problems featuring variable topography. Examples are given for two canonical ocean acoustic environments featuring topography: those of a beach and of an island. Geoacoustic parameters are chosen to be representative of a semihard ocean bottom and are consistent with Miocene/Pliocene sedimentary rocks such as siltstone, shale, and limestone. Reference solutions are generated using a finite-element model [15], [16]. For each example, the reference sound speed is 1500 m/s and environmental parameters are specified in Table I. Tables II and III give range and depth values for the bottom bathymetry/topography. To allow for continuous transmission loss curves that involve plotting in both the water and the sediment, the pressure in the sediment

$$p_b = \frac{\rho_b c_b^2}{2}\Delta_b, \qquad \Delta_b = \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} \tag{5}$$

is calculated from the elastic solution (Δ_b is the dilatation in the sediment) and loss is plotted relative to the pressure in the water, 1 m from the source.

For example A, the variable-rotated solution is applied to a problem featuring a beach 700 m in length and 1800 m distant from the source, a near-surface sediment layer 11 m thick, and another 63-m sediment layer. The environment has been chosen to model a beach at Camp Pendleton, CA, where past experiments have been conducted to investigate near-shore acoustic propagation properties [17]. The 150-Hz source is at 30-m depth below the ocean surface. Source frequency for this problem is chosen to be as high as possible and still permit a finite-element solution. A transmission loss plot of compressional energy for this problem is shown in Fig. 3(a). Energy is trapped in the sediment layer and is propagated up the beach. Modal cutoff occurs in the sediment, and the layering allows energy to propagate into the topography and illustrates the topographic influence on the seismoacoustic field. For the transmission loss curves in Fig. 3(b), the rotated solution (solid curve) is in good agreement with the finite-element solution (dashed curve). Minor differences in the solution may be a result of two-way effects not captured by the one-way parabolic approximation. In Fig. 3(c), the mapped solution (solid curve) is compared against the finite-element solution. Pattern phase errors are evident and the solutions disagree at long ranges.

Example B features an island approximately 10 km wide with a sediment layer 150 m thick. A 5-Hz source is near the bottom interface. A transmission loss plot of compressional energy for this problem is shown in Fig. 4(a). Energy can be seen passing through the island, where it is mostly converted into shear energy. At the fluid–solid interface on the far side of the island, the energy is converted back into compressional energy that then propagates in the water. A small amount of energy is seen along the sediment–sediment interface and at the top of the island 80 m above sea level. Significant energy is evident in the sediment topography. To illustrate the partition of energy that occurs when

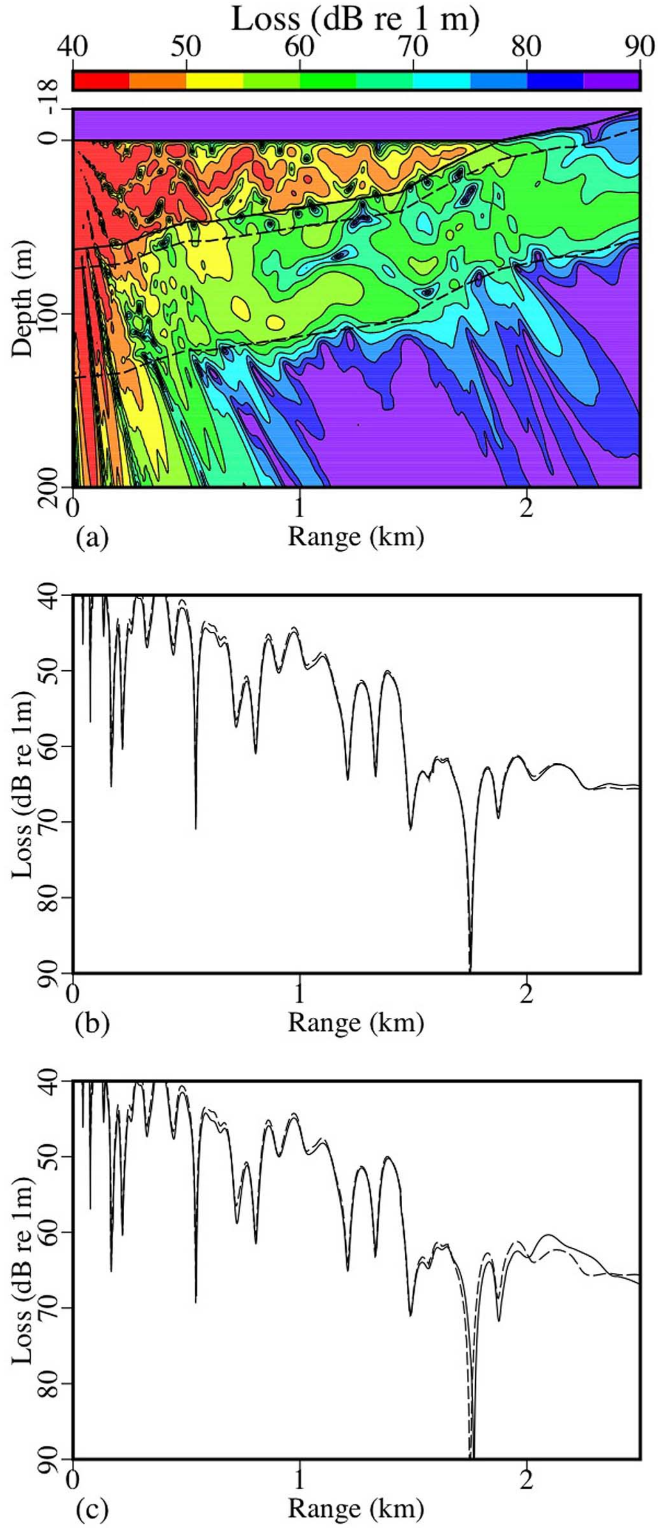

Fig. 3. Compressional wave transmission loss for example A, which models the beach at Camp Pendleton on the southern California coast. The acoustic field is generated by a 150-Hz source at 30-m depth. (a) Contour plot produced by the variable-rotated solution featuring 18 m of topography, an elastic sediment bottom with two near-surface sediment layers, and an elastic half space bottom. (b) and (c) Dashed curves correspond to a reference solution that was generated using a finite-element model. The solid curves are parabolic equation solutions produced by (b) the variable-rotated solution and (c) the mapping solution. Line plots are given for a receiver 30 m below the surface of the ocean.

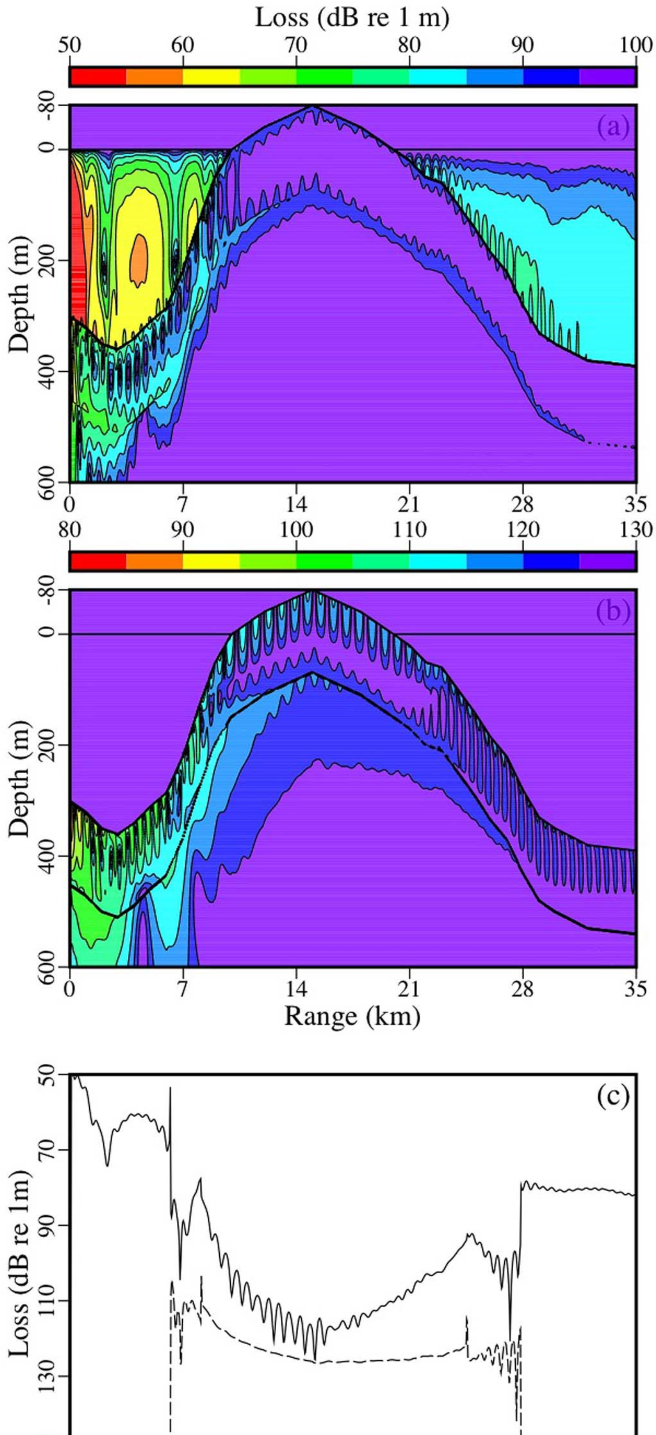

Fig. 4. Transmission loss contours for the (a) compressional and (b) shear fields of example B from the variable-rotated solution. Energy enters the topography and also passes through the 10-km-wide island to reenter the water. (c) Line plots given for a receiver depth of 272 m, where the solid curve is the compressional energy and the dashed curve is the shear.

the acoustic field passes through the island, Fig. 4(b) shows the shear energy for the problem. In Fig. 4(c), line plots are given of the compressional energy (solid curve) and the shear energy (dashed curve) at a receiver depth of 272 m. Benchmark transmission loss curves are not given for this example because coupled mode solutions are currently not capable of solving problems of this type, and finite-element solutions are not given due to memory restrictions on the computer used to generate these results. Other than the mapping solution, we are not aware of another solution method that is capable of solving this problem.

V. Conclusion

The generalized rotated parabolic equation has been extended to solve problems involving variable topography. Air–sediment interface conditions have been implemented to give the required vanishing of normal and tangential stresses at the free surface. Variable topography as well as variable bathymetry is treated by the rotated solution. By comparing the rotated and mapped solutions against normal mode and finite-element solutions, the rotated solution was shown to be capable of handling larger slopes and to be more accurate than the mapped solution. The results presented suggest that the variable-rotated parabolic equation is accurate for a large class of seismoacoustic problems and can provide solutions for important problems not previously accessible; these include propagation up a beach, propagation through an island or seamount, and propagation through a sandbar and then up a beach. The extension to the variable-rotated method presented in this paper makes it the only solution available to accurately and efficiently compute solutions for long-range ocean acoustic problems featuring topography.

Detailed experimental data comparisons for beach and through-island propagation problems are difficult to present, given the combined underwater, and in particular, seismic nature of the problem. A natural extension to this work would be to problems featuring an (elastic) ice cover.

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References

[1] F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, *Computational Ocean Acoustics*. New York: Springer-Verlag, 2000, pp. 325–389.
[2] M. D. Collins, "The rotated parabolic equation and sloping ocean bottoms," *J. Acoust. Soc. Amer.*, vol. 87, pp. 1035–1037, 1990.
[3] M. D. Collins and D. K. Dacol, "A mapping approach for handling sloping interfaces," *J. Acoust. Soc. Amer.*, vol. 107, pp. 1937–1942, 2000.
[4] L. Abrahamsson, L. Andersson, I. Karasalo, and A. Sundstrom, "JEPE—A PE code for range-dependent fluid media," in *Proc. 18th Scandinavian Symp. Phys. Acoust.*, Bergen, Norway, 1995.
[5] I. Karasalo and A. Sundstrom, "JEPE—A high-order PE-model for range-dependent fluid," in *Proc. 3rd Eur. Conf. Underwater Acoust.*, Heraklion, Crete, Greece, 1996, pp. 189–194.
[6] E. Larsson and L. Abrahamsson, "Helmholtz and parabolic equation solutions to a benchmark problem in ocean acoustics," *J. Acoust. Soc. Amer.*, vol. 113, pp. 2446–2454, 2003.
[7] D. A. Outing, W. L. Siegmann, M. D. Collins, and E. K. Westwood, "Generalization of the rotated parabolic equation to variable slopes," *J. Acoust. Soc. Amer.*, vol. 120, pp. 3534–3538, 2006.
[8] M. D. Collins, R. A. Coury, and W. L. Siegmann, "Beach acoustics," *J. Acoust. Soc. Amer.*, vol. 97, pp. 2767–2770, 1995.
[9] D. A. Outing, W. L. Siegmann, and M. D. Collins, "Scholte-to-Rayleigh conversion and other effects in range-dependent elastic media," *IEEE J. Ocean. Eng.*, vol. 32, no. 3, pp. 620–625, Jul. 2007.

[10] M. D. Collins, “An energy-conserving parabolic equation for elastic media,” *J. Acoust. Soc. Amer.*, vol. 94, pp. 975–982, 1993.
[11] R. B. Evans, “A coupled mode solution for acoustic propagation in a waveguide with stepwise depth variations of a penetrable bottom,” *J. Acoust. Soc. Amer.*, vol. 74, pp. 188–195, 1983.
[12] A. J. Fredricks, W. L. Siegmann, and M. D. Collins, “A parabolic equation for anisotropic elastic media,” *Wave Motion*, vol. 31, pp. 139–146, 2000.
[13] W. Jerzak, W. L. Siegmann, and M. D. Collins, “Modeling Rayleigh and Stoneley waves and other interface and boundary effects with the parabolic equation,” *J. Acoust. Soc. Amer.*, vol. 117, pp. 3497–3503, 2005.
[14] M. D. Collins, “A higher-order parabolic equation for wave propagation in an ocean overlying an elastic bottom,” *J. Acoust. Soc. Amer.*, vol. 86, pp. 1459–1464, 1989.
[15] COMSOL Version 3.4, Stockholm, Sweden [Online]. Available: http://www.comsol.com/, last viewed December 1, 2008.
[16] M. Zampolli, A. Tesei, F. B. Jensen, N. Malm, and J. B. Blottman, “A computationally efficient finite element model with perfectly matched layers applied to scattering from axially symmetric objects,” *J. Acoust. Soc. Amer.*, vol. 122, pp. 1472–1485, 2007.
[17] W. S. Hodgkiss, G. L. D’Spain, and W. A. Kuperman, “Characteristics of seismoacoustic propagation in a near-shore environment (A),” *J. Acoust. Soc. Amer.*, vol. 98, p. 2952, 1995.

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