Fluid Mechanics Review for EIT

References:


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EGGN 351 – Introduction to Fluid Mechanics

EQUATION SHEET

[Final Exam]

Acceleration of a fluid particle:
\[ \ddot{a} = \frac{D\ddot{V}}{Dt} = \frac{\partial \ddot{V}}{\partial t} + u \frac{\partial \ddot{V}}{\partial x} + v \frac{\partial \ddot{V}}{\partial y} + w \frac{\partial \ddot{V}}{\partial z} \]

Ideal Gas Law:
\[ p/\rho = RT, \quad \frac{R}{\bar{R}} = \frac{\bar{R}}{M}, \quad \bar{R} = 8.314 \text{ kJ/(kmol.K)} \]

Buoyancy force:
\[ F_B = \gamma_{\text{fluid}}(\text{displaced volume}) \]

Hydrostatics constant density:
\[ p_2 - p_1 = -\gamma(z_2 - z_1) \]
where
\[ \gamma = \rho g \]

Hydrostatic force on planar surface:
\[ F = \gamma h_{CG} A \]
\[ y_{CP} = -\frac{I_{zz'} \sin \theta}{h_{CG} A}, \quad x_{CP} = -\frac{I_{z'z} \sin \theta}{h_{CG} A} \]
where, \( CG \) denotes the centroid of the flat surface, \( \theta \) is the angle between the flat surface and the horizontal, \( I_{zz'} \) is the second moment of area of the flat surface, and \( I_{z'z} \) is the product of area of the flat surface, where the \( \prime \)'s refer to axes going through the centroid. Note that the signs in these equations depend on the direction of the y axis. Here it is assumed that the y axis is directed “upward.”

Reynolds Theorem
\[ \frac{dB}{dt}_{\text{sys}} = \frac{d}{dt} \left( \int_{C.V.} \rho \beta dV \right) + \int_{C.S.} \rho \beta \ddot{V} \cdot d\vec{A} \]
where \( B \) is an extensive variable and \( \beta \) is the corresponding intensive variable.

Steady flow energy equation (one inlet, one outlet)
\[ \left( \frac{p}{\rho g} + \frac{V^2}{2g} + z \right)_1 = \left( \frac{p}{\rho g} + \frac{V^2}{2g} + z \right)_2 + h_{\text{friction}} - h_{\text{pump}} - h_{\text{turbine}} \]

Steady incompressible flow (Bernoulli Equation)
\[ \left( \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_1 = \left( \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_2 = \text{const} \]

Continuity Equation
\[ \frac{\partial p}{\partial t} + \nabla \cdot (\rho \ddot{V}) = 0 \]

Cartesian coordinates
\[ \frac{\partial p}{\partial t} + \frac{\partial p u}{\partial x} + \frac{\partial p v}{\partial y} + \frac{\partial p w}{\partial z} = 0 \]

Navier-Stokes Equations (Incompressible Newtonian fluid)
\[ \frac{D\ddot{V}}{Dt} = -\frac{\ddot{V}}{\rho} + \nu \nabla^2 \ddot{V} + \ddot{g} \]

Cartesian coordinates
\[ \begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \\
&+ \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + g_z \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \\
&+ \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + g_y \\
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \\
&+ \nu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + g_z
\end{align*} \]

Internal Flow

Entrance length, laminar flow:
\[ L_e/D \approx 0.06 \text{ Re}_D \]
PART I
Fluid Properties

The following are some of the common fluid properties you should be familiar with:

- **density** $\rho = \frac{M}{V}$ (11.1.1)
- **specific weight** $\gamma = \rho g = \frac{W}{V}$ (11.1.2)
- **viscosity** $\mu = \frac{\tau}{du/dy}$ (11.1.3)
- **kinematic viscosity** $\nu = \frac{\mu}{\rho}$
- **specific gravity** $SG = \frac{\rho}{\rho_{H_2O}}$
- **bulk modulus** $K = -\frac{\nu \Delta P}{\Delta V}$
- **speed of sound** $c_{\text{liquid}} = \sqrt{K/\rho}$
- **$c_{\text{gas}} = \sqrt{kRT}$** ($k_{\text{air}} = 1.4$)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Definition</th>
<th>Water (20°C, 68°F)</th>
<th>Air (STP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>density</td>
<td>$\rho$</td>
<td>mass/volume</td>
<td>1000 kg/m³</td>
<td>1.23 kg/m³</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.94 slug/ft³</td>
<td>0.0023 slug/ft³</td>
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<tr>
<td>viscosity</td>
<td>$\mu$</td>
<td>shear stress/velocity gradient</td>
<td>$10^{-3}$ N·s/m²</td>
<td>$2.0 \times 10^{-5}$ N·s/m²</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$2 \times 10^{-5}$ lb·sec/ft²</td>
<td>$3.7 \times 10^{-7}$ lb·sec/ft²</td>
</tr>
<tr>
<td>kinematic viscosity</td>
<td>$\nu$</td>
<td>viscosity/density</td>
<td>$10^{-6}$ m²/s</td>
<td>$1.6 \times 10^{-5}$ m²/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$10^{-5}$ ft²/sec</td>
<td>$1.6 \times 10^{-4}$ ft²/sec</td>
</tr>
<tr>
<td>speed of sound</td>
<td>$c$</td>
<td>velocity of propagation of a small wave</td>
<td>1480 m/s</td>
<td>343 m/s</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>4900 ft/sec</td>
<td>1130 ft/sec</td>
</tr>
<tr>
<td>specific weight</td>
<td>$\gamma$</td>
<td>weight/volume</td>
<td>9800 N/m³</td>
<td>12 N/m³</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>62.4 lb/ft³</td>
<td>0.077 lb/ft³</td>
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<tr>
<td>surface tension</td>
<td>$\sigma$</td>
<td>stored energy per unit area</td>
<td>0.073 J/m²</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.005 lb/ft</td>
<td></td>
</tr>
<tr>
<td>bulk modulus</td>
<td>$K$</td>
<td>$\text{-volume} \frac{\Delta \text{pressure}}{\Delta \text{volume}}$</td>
<td>$220 \times 10^4$ kPa</td>
<td>$323,000$ psi</td>
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<tr>
<td>vapor pressure</td>
<td>$p_v$</td>
<td>pressure at which liquid &amp; vapor are in equilibrium</td>
<td>2.45 kPa</td>
<td>0.34 psia</td>
</tr>
</tbody>
</table>
## Dimensions associated with Common Physical Quantities

<table>
<thead>
<tr>
<th></th>
<th>( FLT ) System</th>
<th>( MLT ) System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Acceleration</strong></td>
<td>( LT^{-2} )</td>
<td>( LT^{-2} )</td>
</tr>
<tr>
<td><strong>Angle</strong></td>
<td>( F^0L^0T^0 )</td>
<td>( M^0L^0T^0 )</td>
</tr>
<tr>
<td><strong>Angular acceleration</strong></td>
<td>( T^{-2} )</td>
<td>( T^{-2} )</td>
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<tr>
<td><strong>Angular velocity</strong></td>
<td>( T^{-1} )</td>
<td>( T^{-1} )</td>
</tr>
<tr>
<td><strong>Area</strong></td>
<td>( L^2 )</td>
<td>( L^2 )</td>
</tr>
<tr>
<td><strong>Density</strong></td>
<td>( FL^{-4}T^2 )</td>
<td>( ML^{-3} )</td>
</tr>
<tr>
<td><strong>Energy</strong></td>
<td>( FL )</td>
<td>( ML^2T^{-2} )</td>
</tr>
<tr>
<td><strong>Force</strong></td>
<td>( F )</td>
<td>( MLT^{-2} )</td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
<td>( T^{-1} )</td>
<td>( T^{-1} )</td>
</tr>
<tr>
<td><strong>Heat</strong></td>
<td>( FL )</td>
<td>( ML^2T^{-2} )</td>
</tr>
<tr>
<td><strong>Length</strong></td>
<td>( L )</td>
<td>( L )</td>
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<tr>
<td><strong>Mass</strong></td>
<td>( FL^{-1}T^2 )</td>
<td>( M )</td>
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<tr>
<td><strong>Modulus of elasticity</strong></td>
<td>( FL^{-2} )</td>
<td>( ML^{-1}T^{-2} )</td>
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<tr>
<td><strong>Moment of a force</strong></td>
<td>( FL )</td>
<td>( ML^2T^{-2} )</td>
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<tr>
<td><strong>Moment of inertia (area)</strong></td>
<td>( L^4 )</td>
<td>( L^4 )</td>
</tr>
<tr>
<td><strong>Moment of inertia (mass)</strong></td>
<td>( FLT^2 )</td>
<td>( ML^2 )</td>
</tr>
<tr>
<td><strong>Momentum</strong></td>
<td>( FT )</td>
<td>( MLT^{-1} )</td>
</tr>
<tr>
<td><strong>Power</strong></td>
<td>( FLT^{-1} )</td>
<td>( ML^2T^{-3} )</td>
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<td><strong>Pressure</strong></td>
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<tr>
<td><strong>Specific heat</strong></td>
<td>( L^3T^{-2} )</td>
<td>( L^3T^{-2} )</td>
</tr>
<tr>
<td><strong>Specific weight</strong></td>
<td>( FL^{-3} )</td>
<td>( ML^{-2}T^{-2} )</td>
</tr>
<tr>
<td><strong>Strain</strong></td>
<td>( F^0L^0T^0 )</td>
<td>( M^0L^0T^0 )</td>
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<tr>
<td><strong>Stress</strong></td>
<td>( FL^{-2} )</td>
<td>( ML^{-1}T^{-2} )</td>
</tr>
<tr>
<td><strong>Surface tension</strong></td>
<td>( FL^{-1} )</td>
<td>( MT^{-2} )</td>
</tr>
<tr>
<td><strong>Temperature</strong></td>
<td>( \Theta )</td>
<td>( \Theta )</td>
</tr>
<tr>
<td><strong>Time</strong></td>
<td>( T )</td>
<td>( T )</td>
</tr>
<tr>
<td><strong>Torque</strong></td>
<td>( FL )</td>
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<tr>
<td><strong>Velocity</strong></td>
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<td>( LT^{-1} )</td>
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<tr>
<td><strong>Viscosity (dynamic)</strong></td>
<td>( FL^{-2}T )</td>
<td>( ML^{-1}T^{-1} )</td>
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<tr>
<td><strong>Viscosity (kinematic)</strong></td>
<td>( L^2T^{-1} )</td>
<td>( L^2T^{-1} )</td>
</tr>
<tr>
<td><strong>Volume</strong></td>
<td>( L^3 )</td>
<td>( L^3 )</td>
</tr>
<tr>
<td><strong>Work</strong></td>
<td>( FL )</td>
<td>( ML^2T^{-2} )</td>
</tr>
</tbody>
</table>
Viscosity is defined by $\tau = \mu \frac{du}{dy}$
PART I: FLUID PROPERTIES

Example Problems

- A fluid is a substance that
  a) is essentially incompressible.
  b) always moves when subjected to a shearing stress.
  c) has a viscosity that always increases with temperature.
  d) has a viscosity that always decreases with temperature.
  e) expands until it fills its space.

- Viscosity has dimensions of
  a) $FT^2/L$  b) $F/LT^2$  c) $M/LT^2$  d) $M/LT$  e) $ML/T$

  The viscosity of a fluid varies with:
  a) temperature.  d) temperature and pressure.
  b) pressure.  e) temperature, pressure, and density.
  c) density.

- In an isothermal atmosphere the pressure
  a) is constant with elevation.
  b) decreases linearly with elevation.
  c) cannot be related to elevation.
  d) decreases near the surface but approaches a constant value.
  e) decreases exponentially with elevation.
A torque of 1.6 N·m is needed to rotate the cylinder at 1000 rad/s. Estimate the viscosity (N·s/m²).

a) 0.1  b) 0.2  c) 0.3  
d) 0.4  e) 0.5
Entrance length, turbulent flow:

\[ L_e/D \approx 4.4 \operatorname{Re}_D^{1/6} \]

Darcy-Weisbach:

\[ h_f = \frac{f L V^2}{D^2 g} \]

Laminar flow in a circular pipe:

\[ u(r) = u_{max} \left(1 - \frac{r^2}{R^2}\right) \]

\[ u_{max} = 2V \]

\[ h_f,\text{lam} = \frac{128 \mu L Q}{\pi \rho g D^4} = \frac{32 \mu LV}{\rho g D^2} \]

Colebrook’s formula:

\[ \frac{1}{\sqrt{f}} = -2.0 \log \left( \frac{e/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right) \]

Haaland’s formula:

\[ \frac{1}{\sqrt{f}} \approx -1.8 \log \left( \left( \frac{e/D}{3.7} \right)^{1.11} + \frac{6.0}{Re_D} \right) \]

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External Flows

Drag Coefficient:

\[ C_D = \frac{F_D}{\frac{1}{2} \rho U^2 A} \]

Lift Coefficient:

\[ C_L = \frac{F_L}{\frac{1}{2} \rho U^2 A} \]

Lift-induced drag:

\[ C_D \approx C_{D,\infty} + \frac{C_l^2}{\pi AR} \]

\[ AR = b^2/A_p \] is the aspect ratio. \( b \) is the wing span.

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Obstruction flow meters

\[ \dot{m} = \frac{C_d A_1}{\sqrt{1 - \beta^2}} \sqrt{20(p_1 - p_2)} \]

Note: Assumes no elevation difference between pressure taps.

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Turbomachinery

Net Positive-Suction Head:

\[ \text{NPSH} = \frac{p_h}{\rho g} + \frac{V_i^2}{2g} - \frac{p_v}{\rho g} \]

subscript \( i \) denotes the pump inlet and \( p_v \) is the liquid vapor pressure.

Pump Similarity:

\[ C_Q = \frac{Q}{nD^3} \]

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\[ C_H = \frac{gH}{n^2 D^2} \]

\[ C_P = \frac{\text{bhp}}{\rho m^3 D^6} \]

NOTE: \( n \) is in rps.

Maximum efficiency for similar pumps

Size effect correction (Moody):

\[ \frac{1 - \eta_2}{1 - \eta_1} \approx \left( \frac{D_1}{D_2} \right)^{1/4} \]

Flow-rate effect correction (Anderson):

\[ \frac{0.94 - \eta_2}{0.94 - \eta_1} \approx \left( \frac{Q_1}{Q_2} \right)^{0.32} \]

Specific speed

Rigorous form:

\[ N_s' = \frac{n(Q^*)^{1/2}}{(gH^*)^{3/4}} \]

Common form:

\[ N_s = \frac{(n[\text{rpm}]) (Q^* [\text{gpm}])^{1/2}}{(H^* [\text{ft}])^{3/4}} \]

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Open Channel Flows

The Manning formula:

\[ V_0 \approx \alpha \frac{R_{h}^{2/3}}{S_0^{1/2}} \]

\( \alpha = 1.0 \) in the SI unit system and 1.486 in the BG system; \( n \) is the Manning roughness parameter; \( R_h \) is the hydraulic radius; and \( S_0 \) is the channel slope.

Critical depth (rectangular channel):

\[ y_c = \left( \frac{q}{g} \right)^{1/3} \]

\( q = Q/b \) is the discharge per unit width.

Hydraulic jump:

\[ \frac{y_2}{y_1} = \frac{-1 + \sqrt{1 + 8 Fr_1^2}}{2} \]

\( y_1 \) is the depth before the jump, \( Fr_1 = V_1/\sqrt{g y_1} \) the Froude number before the jump, and \( y_2 \) the depth after the jump.

\[ h_f = \frac{(y_2 - y_1)^3}{4y_1y_2} \]

\( h_f \) is the dissipation head.
Fluid Mechanics Review for EIT

Part I: General...Basic knowledge what a fluid is and basic fluid properties
- Fluid Properties
  - density...ρ = M/V
  - specific weight...γ = W/V
  - viscosity...μ = τ/[du/dy]
  - kinematic viscosity...ν = μ/ρ
  - specific gravity...SG = ρ_1/ρ_H2O
  - vapor pressure...Pressure at which liquid & vapor are in equilibrium

Part II: Fluid Statics...based on the principle dp = γ dh
- manometers....
- forces on plane & curved surfaces
- buoyancy

Part III: Dimensionless Parameters and Similitude
- Reynolds number...inertial force/viscous force...Re = Vlρ/μ = Vd/ν
- Froude number...inertial force/Gravity force...Fr = V^2/lg
- Drag coefficient...drag force/inertial force...C_D = drag/(.5 ρ V^2 A)

Part IV: Control Volume
- continuity...A_1V_1 = A_2V_2
- momentum...∑ F = ρQ(V_2 - V_1)
- energy...± W_s/γg = [V_2^2 - V_1^2]/2g + [p_2 - p_1]/γ + z_2 - z_1 + head losses
- Things to review:
  1. Bernoulli's Equation...[V_2^2 - V_1^2]/2g + [p_2 - p_1]/γ + z_2 - z_1 = 0
  2. Moody Diagram
  3. losses
    - friction factor (Darcy-Weisbach)...h_f = f (L/D) (V^2/2g)
    - minor losses...∑ K_L V^2/2g
  4. hydraulic radius...R_H = A/P

Part V: Open Channel Flow
- Chezy-Manning equation...Q = 1.0/n AR_H^{2/3} S^{1/2}
PART II
Hydrostatics

Typical Problems: Manometers
   Forces on a Plane
   Forces on Curved Surfaces
   Buoyancy

These problems are solved by using the pressure distribution derived from:

\[ dp = \gamma \, dh \]

Or

\[ P = \gamma \, h \text{ if } \gamma \text{ is constant.} \]
Manometers

\[
\begin{align*}
\rho_1 + 0.20 \gamma_W - 0.22 S_1 \gamma_W - 0.13 S_{\text{air}} \gamma_W + 0.13 S_2 \gamma_W &= \rho_5 \\
\rho_1 - \rho_5 &= \gamma_W (-0.20 + 0.22 \times 1.6 - 0.13 \times 0.9)
\end{align*}
\]

where \( \gamma = 9810 \text{ N/m}^3, S_1 = 1.6, S_2 = 0.9, \) and \( S_{\text{air}} = 0. \) Thus

\[
\rho_1 - \rho_5 = 9810(-0.20 + 1.6 \times 0.22 + 0 \times 0.13 + 0.9 \times -0.13)
\]

\[= 343 \text{ Pa} \]
Hydrostatic Forces

Flat Plane:

Hydrostatic force on Plane: \( F = \gamma h_c A \)
Location of force \( y_p = y_c + l_c / (y_c A) \)
PART II: FLUID STATICS

Example Problems

- The pressure $p$, in kPa, is
  a) 51.3
  b) 48.0
  c) 45.2
  d) 40.0
  e) 37.0

- The force $F$, in Newtons, is
  a) 25
  b) 8.9
  c) 2.5
  d) 1.5
  e) 0.36
The force $P$, in kN, to just open the 3-m-wide gate is

a) 55  
b) 60  
c) 65  
d) 70  
e) 75
PART III
Part III. Dimensionless Parameters and Similitude

Similitude and Dimensional Analysis

Laws if similitude make it possible to predict the performance of the prototype from the tests on the model

- Geometric Similarity .... Scale ratio: \( L_r = \frac{L_p}{L_m} \)
- Kinematic Similarity .... \( V_r = \frac{V_p}{V_m} \)
- Dynamic Similarity .... \( F_r = \frac{F_p}{F_m} \)

Important Force Ratio Numbers:

Reynolds Number: Ratio of inertia forces to viscous forces:
\[
R = \frac{L V}{\nu}
\]

Reynolds Number Similarity:
\[
\left( \frac{L V}{\nu} \right)_r = R_r = R_p = \left( \frac{L V}{\nu} \right)_p
\]

Froude Number: Ratio of Inertia and Gravity forces:
\[
F = \frac{V}{\sqrt{gL}}
\]
\[
\left( \frac{V}{\sqrt{gL}} \right)_r = F_r = F_p = \left( \frac{V}{\sqrt{gL}} \right)_p
\]

Note: Used only if have a free surface
Example Problems

- Arrange pressure $p$, flow rate $Q$, diameter $D$, and density $\rho$ into a dimensionless group.
  
a) $pQ^2/\rho D^4$  
b) $p/\rho Q^2 D^4$  
c) $pD^4/\rho Q^2$  
d) $pD^4/\rho Q^2$  
e) $p/\rho Q^2$

- The Reynolds number is a ratio of 
a) velocity effects to viscous effects.  
b) inertial forces to viscous forces.  
c) mass flux to viscosity.  
d) flow rate to kinematic viscosity.  
e) mass flux to kinematic viscosity.
• What flow rate, in m³/s, is needed using a 20:1 scale model of a dam over which 4 m³/s of water flows?
  a) 0.010  b) 0.0068  c) 0.0047  d) 0.0022  e) 0.0015

• It is proposed to model a submarine moving at 10 m/s by testing a 10:1 scale model. What velocity, in m/s, would be needed in the model study?
  a) 1  b) 10  c) 40  d) 80  e) 100
PART IV
Control Volume Equations

Continuity Equation:

\[ \dot{m}_2 = \dot{m}_1 \]

Since the mass flowrate is equal to the product of fluid density, \( \rho \), and volume flowrate, \( Q \), we obtain from Eq. 2

\[ \rho_2 Q_2 = \rho_1 Q_1 \]

Liquid flow at low speeds, as in this example, may be considered incompressible. Therefore

\[ \rho_2 = \rho_1 \]

\[ \dot{Q}_2 = \dot{Q}_1 \]

\[ Q_1 = Q_2 = V_2 A_2 \]

Momentum Equation:

\[ \frac{\partial}{\partial t} \int_{cv} \dot{V} \rho \, dV + \int_{cs} \dot{V} \rho \dot{V} \cdot \mathbf{n} \, dA = \sum_{\text{contents of the control volume}} F \]

\[ \sum F = \rho_2 A_2 V_2 V_2 - \rho_1 A_1 V_1 V_1 \]

Energy Equation:

\[ \frac{P_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + g z_{\text{out}} = \frac{P_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + g z_{\text{in}} + w_{\text{shaft net in}} - \text{loss} \]

\[ -23 \quad \left( \frac{P}{\rho} + \frac{V^2}{2g} + Z \right)_{\text{in}} = \left( \frac{P}{\rho} + \frac{V^2}{2g} + Z \right)_{\text{out}} + h_f - h_p + h_t \]
Energy Grade Line

Hydraulic grade line (HGL) and energy grade line (EGL) for a piping system.

When the energy equation is written in the form

\[
\left( \frac{P}{\rho g} + \frac{V^2}{2g} + e \right)_1 = \left( \frac{P}{\rho g} + \frac{V^2}{2g} + e \right)_2 + h_f - h_i + h
\]

the terms have the dimension of length (head form of the energy equation).

**Hydraulic grade line (HGL):** formed by the locus of points located a distance of p/\gamma above the center of the pipe. The liquid in a piezometer would rise to this level.

**Energy grade line (EGL):** formed by the locus of points a distance \(V^2/2g\) above the HGL. The liquid in a pitot tube would rise to this level.
(Momentum)

If the density of the air is 1.2 kg/m³, find $F$, in Newtons.

a) 2.4
b) 3.6
c) 4.8
d) 7.6
e) 9.6

What force, in Newtons, acts on the nozzle?

8 cm dia

4 cm dia

water

800 kPa

2 cm dia

air

$V = 80$ m/s

a) 4020  b) 3230  c) 2420  d) 1830  e) 161
PART IV: Control Volume Problems

Example Problems

• (Continuity)

The velocity in a 2-cm-dia pipe is 20 m/s. If the pipe enlarges to 5-cm-dia, the velocity, in m/s, will be

a) 8.0  

b) 6.4  

c) 5.2  

d) 4.8  

e) 3.2

• (Bernoulli's)

Calculate V, in m/s.

a) 8  

b) 7  

c) 6  

d) 5  

e) 4
The Moody Diagram.
Losses in a Developed Pipe Flow

Head loss that results from the wall shear in a developed flow (Darcy-Weisbach):

\[ \frac{\Delta p}{\gamma} = h_L = f \frac{L}{D} \frac{V^2}{2g} \]

Where

\[ f = \frac{64}{Re} \quad \text{Re} < 2100 \]

\[ \frac{1}{f^{1/2}} = -2.0 \log \left( \frac{e/d}{3.7} + \frac{2.51}{Re_d f^{1/2}} \right) \]
• (Losses)

In a completely turbulent flow the head loss
a) increases with the velocity.         d) increases with diameter.
b) increases with the velocity squared. e) increases with flow rate.
c) decreases with wall roughness.

• (Losses)

The shear stress in a turbulent pipe flow
a) varies parabolically with the radius.
b) is constant over the pipe radius.
c) varies according to the 1/7th power law.
d) is zero at the center and increases linearly to the wall.
e) is zero at the wall and increases linearly to the center.

• (Losses)

The head loss in a pipe flow can be calculated using
a) the Bernoulli equation.         d) the Momentum equation.
b) Darcy's law.                             e) the Darcy-Weisbach equation.
c) the Chezy-Manning equation.
(Losses)

The velocity distribution in a turbulent flow in a pipe is often assumed to a) vary parabolically.
b) be zero at the wall and increase linearly to the center.
c) vary according to the 1/7th power law.
d) be unpredictable and is thus not used.
e) be maximum at the wall and decrease linearly to the center.

The Moody diagram is sketched. The friction factor for turbulent flow in a smooth pipe is given by curve

- (Losses)

Water flows through a 10-cm-dia, 100-m-long pipe connecting two reservoirs with an elevation difference of 40 m. The average velocity is 6 m/s. Neglecting minor losses, the friction factor is

- a) 0.020  - b) 0.022  - c) 0.024  - d) 0.026  - e) 0.028
Find the energy required, in kW, by the 85% efficient pump if \( Q = 0.02 \text{ m}^3/\text{s} \).

a) 14  b) 20  c) 28  d) 35  e) 44
PART V
Chezy-Manning Equation

\[ Q = \left( \frac{k}{n} \right) A R_h^{2/3} S^{1/2} \]

where

\( R_h = \text{hydraulic radius} = \text{area/wetted perimeter} \)

\( A = \text{cross-sectional area} \)

\( n = \text{Manning number obtained from table below} \)

\( k = 1.0 \text{ if SI units or } 1.49 \text{ if English units} \)

<table>
<thead>
<tr>
<th>Wall Material</th>
<th>Manning n</th>
<th>Wall Material</th>
<th>Manning n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planed wood</td>
<td>.012</td>
<td>Concrete pipe</td>
<td>.015</td>
</tr>
<tr>
<td>Unplaned wood</td>
<td>.013</td>
<td>Riveted steel</td>
<td>.017</td>
</tr>
<tr>
<td>Finished concrete</td>
<td>.012</td>
<td>Earth, straight</td>
<td>.022</td>
</tr>
<tr>
<td>Unfinished concrete</td>
<td>.014</td>
<td>Corrugated metal flumes</td>
<td>.025</td>
</tr>
<tr>
<td>Sewer Pipe</td>
<td>.013</td>
<td>Rubble</td>
<td>.03</td>
</tr>
<tr>
<td>Brick</td>
<td>.016</td>
<td>Earth with stones and weeds</td>
<td>.035</td>
</tr>
<tr>
<td>Cast iron, wrought iron</td>
<td>.015</td>
<td>Mountain streams</td>
<td>.05</td>
</tr>
</tbody>
</table>
1. Flow through a sudden contraction in a pipe
   (a) is usually laminar
   (b) results in a significant energy loss
   (c) can be analyzed using Bernoulli's equation
   (d) has a relatively small loss coefficient
   (e) does not tend to separate

2. The dimensions of dynamic viscosity \( \mu \) are
   (a) \( ML^{-1}T^{-1} \)  
   (b) \( M^{1.5}L^{-1}T^{-1} \)  
   (c) \( M^{1}L^{-1}T^{-1} \)  
   (d) \( M^{-1}L^{-1}T \)  
   (e) \( M^{1}L^{-1}T^{1} \)

3. The pressure loss due to friction in a horizontal section of constant diameter pipe is usually determined by
   (a) continuity equation  
   (b) Chezy-Manning equation  
   (c) Bernoulli's equation  
   (d) Darcy equation  
   (f) Navier-Stokes equation

4. Flow over a 27-m high dam is to be studied in a lab with a 3-m-high model. If the river has a flow rate of 74 m\(^3\)/s, the model flow-rate should be
   (a) 30 m\(^3\)/s  
   (b) 3 m\(^3\)/s  
   (c) 0.3 m\(^3\)/s  
   (d) 9 m\(^3\)/s  
   (e) 0.9 m\(^3\)/s

5. Consider the jet of water striking the plate. Assume that \( A = 0.02 \text{ m}^2 \), \( \rho = 1000 \text{ kg/m}^3 \), and \( V = 20 \text{ m/s} \).

   \[ \text{At a plate radius of 20 cm, the fluid thickness } t \text{ is about} \]
   (a) 8 mm  
   (b) 16 mm  
   (c) 30 mm  
   (d) 52 mm  
   (e) 85 mm

6. The force \( P \) required per meter of gate width (into the paper) is
   (a) 29 kN  
   (b) 33 kN  
   (c) 35 kN  
   (d) 103 kN  
   (e) 980 kN