



God doesn't play dice -- Albert Einstein

Einstein's famous quotation was not about his speculations concerning the gambling propensities of God, but rather an expression of his dissatisfaction with the apparently probabilistic description of nature embodied by the quantum theory.

Suppose God did favor games of chance, and that he opened two casinos: the Classical Casino, and the Quantum Casino. In both casinos, craps is the game played, where two independent dice are rolled, and (for simplicity here), a winning roll has a total of seven, while a losing roll has a total of two ("snake eyes" or "craps").

In the Classical Casino, the laws of probability are the classical ones, while in the Quantum Casino, the die are indistinguishable, and for this particular homework, they obey Fermi statistics.

Warm up before entering the casinos:

- 1) For the Classical Casino, how many possible different rolls (states of the combined system of two die) are there, if the dice are distinguishable? How many possible rolls are there for the two Fermionic dice?
- 2) What is the probability of winning and losing in the Classical Casino? In the Quantum Casino?

Now, suppose that Lucifer has managed to substitute the dice in each casino with "loaded dice". In particular, each die has an off-center weight inside which causes the energy of the roll "one" to be lower than the average by $\Delta E = -\Delta(mgh)$, while a "six" has an energy which is higher than the average by $\Delta E = \Delta(mgh)$. (Take the zero of energy to be the energy of all the other rolls.)

Questions:

- 1) What is the partition function for the two loaded dice in the Classical Casino? In the Quantum Casino?
- 2) In which Casino are the odds better for winning? Does it matter whether the dice are "cold", $k_B T \ll \Delta(mgh)$, or "hot" $k_B T \gg \Delta(mgh)$? Calculate the odds in the two limits of $T=0$ and $T = \text{infinity}$.

There are only two ways to live your life. One is as though nothing is a miracle. The other is as though everything is a miracle. -- Albert Einstein