Retrieving the Green's function in an open system by cross correlation: A comparison of approaches (L)

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We compare two approaches for deriving the fact that the Green's function in an arbitrary inhomogeneous open system can be obtained by cross correlating recordings of the wave field at two positions. One approach is based on physical arguments, exploiting the principle of time-reversal invariance of the acoustic wave equation. The other approach is based on Rayleigh's reciprocity theorem. Using a unified notation, we show that the result of the time-reversal approach can be obtained as an approximation of the result of the reciprocity approach. © *2005 Acoustical Society of America.* [DOI: 10.1121/1.2046847]

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I. INTRODUCTION

Since the work of Weaver and Lobkis,[1,2] many researchers have shown theoretically and experimentally that the cross correlation of the recordings of a diffuse wave field at two receiver positions yields the Green's function between these positions. In most cases it is assumed that the diffuse wave field consists of normal modes (with uncorrelated amplitudes) in a closed system. Less attention has been paid to the theory of Green's function retrieval in arbitrary inhomogeneous open systems. Nevertheless, the first result stems from 1968, albeit for one-dimensional media, when Claerbout[3] showed that the seismic reflection response of a horizontally layered earth can be synthesized from the autocorrelation of its transmission response. Recently we generalized this to three-dimensional (3D) arbitrary inhomogeneous media.[4–6] Using reciprocity theorems of the correlation type, we showed in those papers that the cross correlation of transmission responses observed at the earth's free surface, due to uncorrelated noise sources in the subsurface, yields the full reflection response (i.e., the ballistic wave and the coda) of the 3D inhomogeneous subsurface. Weaver and Lobkis[7] followed a similar approach for a configuration in which the 3D inhomogeneous medium is surrounded by uncorrelated sources. Independently, Derode *et al.*[8,9] derived expressions for Green's function retrieval in open systems using physical arguments, exploiting the principle of time-reversal invariance of the acoustic wave equation. Their approach can be seen as the "physical counterpart" of our derivations based on reciprocity. In this letter we compare the time-reversal approach of Derode *et al.*[8,9] with our approach based on Rayleigh's reciprocity theorem.[4–6] Using a unified notation, we show that the result of the time-reversal approach can be obtained as an approximation of the result of the reciprocity approach.

It should be noted that in both derivations we consider the situation of impulsive point sources, uniformly distributed over a surface surrounding the configuration. We briefly indicate how the resulting expressions can be modified for the situation of uncorrelated noise sources.

II. TIME-REVERSAL APPROACH

In this section we summarize the time-reversal approach of Derode *et al.*[8,9] for deriving expressions for Green's function retrieval. Consider a lossless arbitrary inhomogeneous acoustic medium in a homogeneous embedding. In this configuration we define two points with coordinate vectors $\mathbf{x}_A$ and $\mathbf{x}_B$. Our aim is to show that the acoustic response at $\mathbf{x}_B$ due to an impulsive source at $\mathbf{x}_A$ [i.e., the Green's function $G(\mathbf{x}_B,\mathbf{x}_A,t)$] can be obtained by cross correlating passive measurements of the wave fields at $\mathbf{x}_A$ and $\mathbf{x}_B$ due to sources on a surface S in the homogeneous embedding. The derivation starts by considering another physical experiment, namely an impulsive source at $\mathbf{x}_A$ and receivers at $\mathbf{x}$ on S. The response at one particular point $\mathbf{x}$ on S is denoted by $G(\mathbf{x},\mathbf{x}_A,t)$. Imagine that we record this response for all $\mathbf{x}$ on S, revert the time axis, and feed these time-reverted functions $G(\mathbf{x},\mathbf{x}_A,-t)$ to sources at all $\mathbf{x}$ on S. The superposition principle states that the wave field at any point $\mathbf{x}'$ in S due to these sources on S is then given by

$$u(\mathbf{x}',t) \propto \oint_S \underbrace{G(\mathbf{x}',\mathbf{x},t)}_{\text{'propagator'}} * \underbrace{G(\mathbf{x},\mathbf{x}_A,-t)}_{\text{'source'}} d^2\mathbf{x}, \tag{1}$$

where $*$ denotes convolution and $\propto$ "proportional to." According to this equation, $G(\mathbf{x}',\mathbf{x},t)$ propagates the source function $G(\mathbf{x},\mathbf{x}_A,-t)$ from $\mathbf{x}$ to $\mathbf{x}'$ and the result is integrated over all sources on S. Due to the invariance of the acoustic wave equation for time-reversal, the wave field $u(\mathbf{x}',t)$ focuses for $\mathbf{x}'=\mathbf{x}_A$ at $t=0$. McMechan[10] exploited this property in a seismic imaging method which has become known as reverse time migration. Derode *et al.*[8,9] give a new interpretation to Eq. (1). Since $u(\mathbf{x}',t)$ focuses for $\mathbf{x}'=\mathbf{x}_A$ at $t=0$, the

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wave field $u(\mathbf{x}',t)$ for arbitrary $\mathbf{x}'$ and t can be seen as the response of a virtual source at $\mathbf{x}_A$ and $t=0$. This virtual source response, however, consists of a causal and an anticausal part, according to

$$u(\mathbf{x}',t) = G(\mathbf{x}',\mathbf{x}_A,t) + G(\mathbf{x}',\mathbf{x}_A,-t). \tag{2}$$

This is explained as follows: the wave field generated by the anticausal sources on S first propagates to all $\mathbf{x}'$ where it gives an anticausal contribution, next it focuses in $\mathbf{x}_A$ at $t=0$, and finally it propagates again to all $\mathbf{x}'$ giving the causal contribution. The propagation paths from $\mathbf{x}'$ to $\mathbf{x}_A$ are the same as those from $\mathbf{x}_A$ to $\mathbf{x}'$, but are traveled in opposite direction. Combining Eqs. (1) and (2), applying source-receiver reciprocity to $G(\mathbf{x},\mathbf{x}_A,-t)$ in Eq. (1), and setting $\mathbf{x}'=\mathbf{x}_B$ yields

$$G(\mathbf{x}_B,\mathbf{x}_A,t) + G(\mathbf{x}_B,\mathbf{x}_A,-t) \propto \oint_S G(\mathbf{x}_B,\mathbf{x},t) * G(\mathbf{x}_A,\mathbf{x},-t)d^2\mathbf{x}. \tag{3}$$

The right-hand side of Eq. (3) can be interpreted as the integral of cross correlations of observations of wave fields at $\mathbf{x}_B$ and $\mathbf{x}_A$, respectively, due to impulsive sources at $\mathbf{x}$ on S; the integration takes place along the source coordinate $\mathbf{x}$. The left-hand side is interpreted as the superposition of the response at $\mathbf{x}_B$ due to an impulsive source at $\mathbf{x}_A$ and its time-reversed version. Since the Green's function $G(\mathbf{x}_B,\mathbf{x}_A,t)$ is causal, it can be obtained from the left-hand side of Eq. (3) by taking the causal part. The reconstructed Green's function contains the ballistic wave as well as the coda due to multiple scattering in the inhomogeneous medium.

III. RECIPROCITY APPROACH

In this section we summarize our derivation based on Rayleigh's reciprocity theorem.[4–6] A reciprocity theorem relates two independent acoustic states in one and the same domain.[11,12] Consider an acoustic wave field, characterized by the acoustic pressure $p(\mathbf{x},t)$ and the particle velocity $v_i(\mathbf{x},t)$. We define the temporal Fourier transform of a space- and time-dependent quantity $p(\mathbf{x},t)$ as $\hat{p}(\mathbf{x},\omega) = \int \exp(-j\omega t)p(\mathbf{x},t)dt$, where j is the imaginary unit and ω the angular frequency. In the space-frequency domain the acoustic pressure and particle velocity in a lossless arbitrary inhomogeneous acoustic medium obey the equation of motion $j\omega\rho\hat{v}_i + \partial_i\hat{p}=0$ and the stress-strain relation $j\omega\kappa\hat{p}+\partial_i\hat{v}_i = \hat{q}$, where ∂_i is the partial derivative in the x_i direction (Einstein's summation convention applies for repeated lowercase subscripts), $\rho(\mathbf{x})$ the mass density of the medium, $\kappa(\mathbf{x})$ its compressibility, and $\hat{q}(\mathbf{x},\omega)$ a source distribution in terms of volume injection rate density. We introduce two independent acoustic states, which will be distinguished by subscripts A and B, and consider the following combination of wave fields in both states: $\hat{p}_A\hat{v}_{i,B} - \hat{v}_{i,A}\hat{p}_B$. Note that these products in the frequency domain correspond to convolutions in the time domain. Rayleigh's reciprocity theorem is obtained by applying the differential operator ∂_i, according to $\partial_i\{\hat{p}_A\hat{v}_{i,B} - \hat{v}_{i,A}\hat{p}_B\}$, substituting the equation of motion and the stress-strain relation for states A and B, integrating the result over a spatial domain V enclosed by S with outward pointing normal vector $\mathbf{n}=(n_1,n_2,n_3)$ and applying the theorem of Gauss. This gives

$$\int_V \{\hat{p}_A\hat{q}_B - \hat{q}_A\hat{p}_B\}d^3\mathbf{x} = \oint_S \{\hat{p}_A\hat{v}_{i,B} - \hat{v}_{i,A}\hat{p}_B\}n_i d^2\mathbf{x}. \tag{4}$$

Since the medium is lossless, we can apply the principle of time-reversal invariance.[13] In the frequency domain time-reversal is replaced by complex conjugation. Hence, when $\hat{p}$ and $\hat{v}_i$ are a solution of the equation of motion and the stress-strain relation with source distribution $\hat{q}$, then $\hat{p}^*$ and $-\hat{v}_i^*$ obey the same equations with source distribution $-\hat{q}^*$ (the asterisk denotes complex conjugation). Making these substitutions for state A we obtain

$$\int_V \{\hat{p}_A^*\hat{q}_B + \hat{q}_A^*\hat{p}_B\}d^3\mathbf{x} = \oint_S \{\hat{p}_A^*\hat{v}_{i,B} + \hat{v}_{i,A}^*\hat{p}_B\}n_i d^2\mathbf{x}. \tag{5}$$

Next we choose impulsive point sources in both states, according to $\hat{q}_A(\mathbf{x},\omega)=\delta(\mathbf{x}-\mathbf{x}_A)$ and $\hat{q}_B(\mathbf{x},\omega)=\delta(\mathbf{x}-\mathbf{x}_B)$, with $\mathbf{x}_A$ and $\mathbf{x}_B$ both in V. The wave field in state A can thus be expressed in terms of a Green's function, according to

$$\hat{p}_A(\mathbf{x},\omega) = \hat{G}(\mathbf{x},\mathbf{x}_A,\omega), \tag{6}$$

$$\hat{v}_{i,A}(\mathbf{x},\omega) = -(j\omega\rho(\mathbf{x}))^{-1}\partial_i\hat{G}(\mathbf{x},\mathbf{x}_A,\omega), \tag{7}$$

where $\hat{G}(\mathbf{x},\mathbf{x}_A,\omega)$ obeys the wave equation

$$\partial_i(\rho^{-1}\partial_i\hat{G}) + (\omega^2/\rho c^2)\hat{G} = -j\omega\delta(\mathbf{x}-\mathbf{x}_A), \tag{8}$$

with propagation velocity $c(\mathbf{x})=\{\kappa(\mathbf{x})\rho(\mathbf{x})\}^{-1/2}$; similar expressions hold for the wave field in state B. Substituting these expressions into Eq. (5) and using source-receiver reciprocity of the Green's functions gives

$$2\Re\{\hat{G}(\mathbf{x}_B,\mathbf{x}_A,\omega)\} = \oint_S \frac{-1}{j\omega\rho(\mathbf{x})}(\partial_i\hat{G}(\mathbf{x}_B,\mathbf{x},\omega)\hat{G}^*(\mathbf{x}_A,\mathbf{x},\omega) - \hat{G}(\mathbf{x}_B,\mathbf{x},\omega)\partial_i\hat{G}^*(\mathbf{x}_A,\mathbf{x},\omega))n_i d^2\mathbf{x}, \tag{9}$$

where $\Re$ denotes the real part. Note that the left-hand side is the Fourier transform of $G(\mathbf{x}_B,\mathbf{x}_A,t)+G(\mathbf{x}_B,\mathbf{x}_A,-t)$; the products $\partial_i\hat{G}\hat{G}^*$, etc., on the right-hand side correspond to cross correlations in the time domain. Expressions like the right-hand side of this equation have been used by numerous researchers (including the authors) for seismic migration in the frequency domain. Esmersoy and Oristaglio[14] explained the link with the reverse time migration method, mentioned in Sec. II. What is new (compared with migration) is that Eq. (9) is formulated in such a way that it gives an exact representation of the Green's function $\hat{G}(\mathbf{x}_B,\mathbf{x}_A,\omega)$ in terms of cross correlations of observed wave fields at $\mathbf{x}_B$ and $\mathbf{x}_A$. Note that, unlike in Sec. II, we have not assumed that the medium outside surface S is homogeneous. The terms $\hat{G}$ and $\partial_i\hat{G}$ under the integral represent responses of monopole and dipole sources at $\mathbf{x}$ on S; the combination of the two correlation products under the integral ensures that waves propagating outward from the sources on S do not interact with those propagating inward and vice versa. When a part of S is a free surface on which the acoustic pressure vanishes, then the surface integral in Eq. (5) and hence in Eq. (9) need only be evaluated over the remaining part of S. Other modifications of Eq. (9), including the elastodynamic generalization, are discussed in Refs. 4–6. Van Manen and Robertsson[15] propose an efficient modeling scheme, based on an expression similar to Eq. (9).

Note that for the derivation of expressions (3) and (9) we assumed that impulsive point sources were placed on the surface S. This is the approach taken, e.g., by Bakulin and Calvert[16] in their experiment on virtual source imaging. Our derivation also holds for uncorrelated stationary noise sources on S whose source-time function satisfies $\langle N(\mathbf{x},t)*N(\mathbf{x}',-t)\rangle=\delta(\mathbf{x}-\mathbf{x}')C(t)$, where $\langle\cdot\rangle$ denotes a spatial ensemble average and $C(t)$ the autocorrelation of the noise (which is assumed to be the same for all sources). When the noise is distributed over the surface, the cross-correlation of the observations at $\mathbf{x}_A$ and $\mathbf{x}_B$ leads to a double surface integral. The delta function reduces this to the single surface integral in the theory presented here.[4–7,9,17] A further discussion is beyond the scope of this letter.

IV. COMPARISON

Equation (9) is an exact representation of the real part of the Green's function $\hat{G}(\mathbf{x}_B,\mathbf{x}_A,\omega)$. In comparison with Eq. (3), the right-hand side of Eq. (9) contains two correlation products instead of one. Moreover, each of the correlation products in Eq. (9) involves a monopole and a dipole response instead of two monopole responses. Last but not least, Eq. (9) is formulated in the frequency domain and Eq. (3) in the time domain.

First we discuss how we can combine the two correlation products in Eq. (9) into a single term. To this end we assume that the medium outside S is homogeneous, with constant propagation velocity c and mass density ρ. In the high frequency regime, the derivatives of the Green's functions can be approximated by multiplying each constituent (direct wave, scattered wave, etc.) by $-j(\omega/c)|\cos\alpha|$, where α is the angle between the pertinent ray and the normal on S. The main contributions to the integral in Eq. (9) come from stationary points on S.[17–19] At those points the ray angles for both Green's functions are identical (see also the example in Sec. V). This implies that the contributions of the two terms under the integral in Eq. (9) are approximately equal (but opposite in sign), hence

$$2\Re\{\hat{G}(\mathbf{x}_B,\mathbf{x}_A,\omega)\}\approx\frac{-2}{j\omega\rho}\oint_S\partial_i\hat{G}(\mathbf{x}_B,\mathbf{x},\omega)\hat{G}^*(\mathbf{x}_A,\mathbf{x},\omega)n_i d^2\mathbf{x}. \tag{10}$$

The accuracy of this approximation is demonstrated with a numerical example in Sec. V.

Our next aim is to express the dipole response $n_i\partial_i\hat{G}$ in terms of the monopole response $\hat{G}$. As explained earlier, this could be done by multiplying each constituent by $-j(\omega/c)|\cos\alpha|$. However, since α may have multiple values and since these values are usually unknown (unless the in-

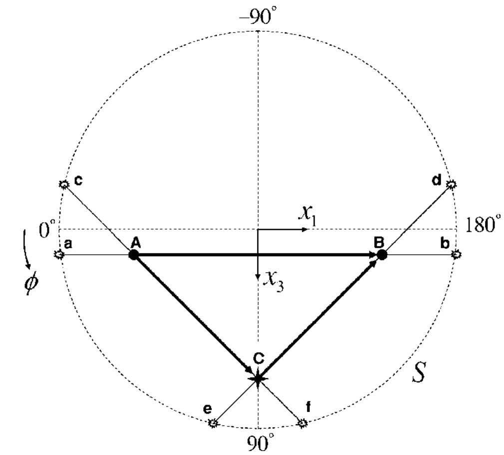

FIG. 1. Single point diffractor (C) in a homogeneous model. The receivers are at A and B. The numerical integration is carried out along the sources on the surface S. The main contributions come from the stationary points a–d. The contributions from stationary points e and f cancel.

homogeneous medium as well as the source positions are accurately known), we approximate $n_i\partial_i\hat{G}$ by $-j(\omega/c)\hat{G}$, hence

$$2\Re\{\hat{G}(\mathbf{x}_B,\mathbf{x}_A,\omega)\}\approx\frac{2}{\rho c}\oint_S\hat{G}(\mathbf{x}_B,\mathbf{x},\omega)\hat{G}^*(\mathbf{x}_A,\mathbf{x},\omega)d^2\mathbf{x}. \tag{11}$$

This approximation is quite accurate when S is a sphere with very large radius so that all rays are normal to S (i.e., $\alpha\approx 0$). In general, however, this approximation involves an amplitude error that can be significant, see the numerical example in Sec. V. However, since this approximation does not affect the phase it is considered acceptable for many practical situations. Transforming both sides of Eq. (11) back to the time domain yields Eq. (3) (i.e., the result of Derode *et al.*[8,9]), with proportionality factor $2/\rho c$.

V. NUMERICAL EXAMPLE

We illustrate Eq. (10) with a simple example. We consider a two-dimensional configuration with a single point diffractor at $(x_1,x_3)=(0,600)$m in a homogeneous medium with propagation velocity $c=2000$ m/s, see Fig. 1, in which C denotes the diffractor. Further, we define $\mathbf{x}_A=(-500,100)$m and $\mathbf{x}_B=(500,100)$m, denoted by A and B in Fig. 1. The surface S is a circle with its center at the origin and a radius of 800 m. The solid arrows in Fig. 1 denote the propagation paths of the Green's function $G(\mathbf{x}_B,\mathbf{x}_A,t)$. For the Green's functions in Eq. (10) we use analytical expressions, based on the Born approximation (hence, the contrast at the point diffractor is assumed to be small). To be consistent with the Born approximation, in the cross correlations we also consider only the zeroth- and first-order terms. Figure 2(a) shows the time-domain representation of the integrand of Eq. (10), convolved with a wavelet with a central frequency of 50 Hz. Each trace corresponds to a fixed source position $\mathbf{x}$ on S; the source position in polar coordinates is $(\phi,r=800)$. The sum of all these traces (multiplied by $rd\phi$) is shown in Fig. 2(b). This result accurately matches the time-domain version of the left-hand side of Eq. (10), i.e., $G(\mathbf{x}_B,\mathbf{x}_A,t)+G(\mathbf{x}_B,\mathbf{x}_A,-t)$, convolved with a wavelet, see

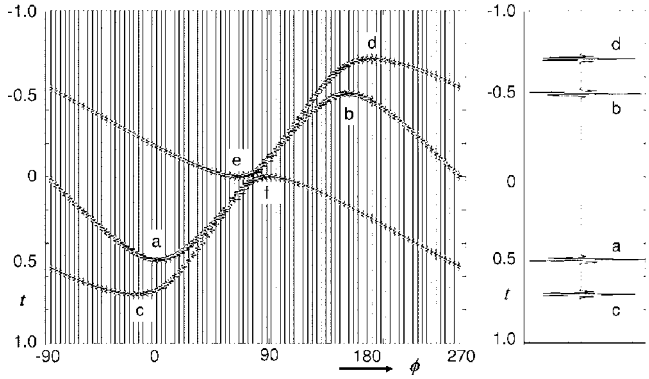

FIG. 2. (a) Time domain representation of the integrand of Eq. (10). (b) The sum of all traces in (a).

Fig. 3. The events labeled "a" and "c" in Fig. 2 are the direct and scattered arrivals; the events "b" and "d" are the corresponding anticausal arrivals. This figure clearly shows that the main contribution to these events come from Fresnel zones around the stationary points of the integrand.[17–19] The sources at these stationary points are marked in Fig. 1 with the same labels. We discuss event "c" in more detail. The path "cCB" in Fig. 1 represents the scattered wave in $G(\mathbf{x}_B,\mathbf{x},t)$, for $\mathbf{x}$ at the stationary point "c." The path "cA" represents the direct wave in $G(\mathbf{x}_A,\mathbf{x},t)$. By correlating these two waves, the travel time along the path "cA" is subtracted from that along the path "cCB," leaving the travel time along the path "ACB," which corresponds to the travel time of the scattered wave in $G(\mathbf{x}_B,\mathbf{x}_A,t)$. This correlation result is indicated by "c" in Fig. 2(a) and the integral over the Fresnel zone around this point is event "c" in Fig. 2(b). The other events in Fig. 2(b) can be explained in a similar way. Finally, note that there are two more stationary points, indicated by "e" and "f" in Figs. 1 and 2(a), of which the contributions cancel each other.

The numerical evaluation of Eq. (11) for the same configuration yields the result represented by the dashed curve in Fig. 3. We observe that the travel time of the scattered wave is accurately captured by this equation, but the amplitude is overestimated (apparently the assumption $\alpha \approx 0$ is not fulfilled here). By increasing the radius of S to 10 000 m we obtained a result with Eq. (11) that again accurately matches the directly modeled wave field (not shown).

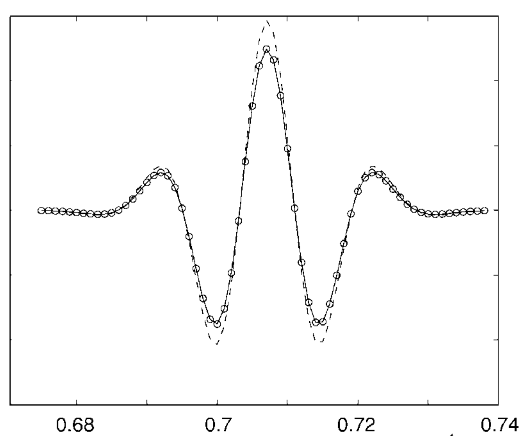

FIG. 3. Zoomed-in version of event c in Fig. 2(b). The solid line is the time-domain version of the left-hand side of Eq. (10). The circles represent the numerical integration result of the right-hand side of Eq. (10) [i.e., the sum of the traces in Fig. 2(a)]. The dashed line represents the numerical integration result of Eq. (11).

VI. CONCLUSIONS

In the literature several derivations have been proposed for Green's function retrieval from cross correlations of wave fields in inhomogeneous open systems. In this letter we compared a derivation based on the time-reversal approach[8,9] with one based on Rayleigh's reciprocity theorem.[4–6] One of the conclusions is that the expression obtained by the time-reversal approach is an approximation of that based on Rayleigh's reciprocity theorem.

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