Cancellation of spurious arrivals in Green's function retrieval of multiple scattered waves

Roel Snieder[a] and Clement Fleury
Center for Wave Phenomena, Colorado School of Mines, Golden, Colorado 80401

(Received 24 March 2010; revised 22 June 2010; accepted 1 August 2010)

The Green's function for wave propagation can be extracted by cross-correlating field fluctuations excited on a closed surface that surrounds the employed receivers. This study treats an acoustic multiple scattering medium with discrete scatterers and shows that for a given source the cross-correlation of waves propagating along most combinations of scattering paths gives unphysical arrivals. Because theory predicts that the true Green's function is retrieved, such unphysical arrivals must cancel after integration over all sources. This cancellation occurs because the scattering amplitude of each scatterer satisfies the generalized optical theorem. The cross-correlation of scattered waves with themselves does not lead to the correct retrieval of scattered waves, because the cross-terms between the direct and scattered waves is essential.
 [DOI: 10.1121/1.3483722]

PACS number(s): 43.20.Fn, 43.20.Bi, 43.30.Ft [RMW] Pages: 1598–1605

I. INTRODUCTION

The extraction of the Green's function for wave propagation by correlation of field fluctuations is an active area of research in a variety of different fields that has reached the stage where material is documented in review papers and books.[1–7] The central idea is that field fluctuations recorded at two points lead after cross-correlation to the superposition of the causal and time-reversed Green's function for wave propagation between those points. This principle finds its roots in the fluctuation dissipation theorem,[8,9] and was known for a considerable time for electrical systems.[10,11] This principle has recently been extended to a general class of physical fields,[12–15] including static fields.[16,17] Green's function retrieval for the acoustic waves treated here is based on the cross-correlation of field fluctuations that are excited by sources with equal power spectrum that are located on a closed surface surrounding the used receivers.[18,19] When these sources are located on a spherical surface ∂V where the waves satisfy a radiation boundary condition, the principle of Green's function extraction for acoustic waves is, in the frequency domain, formulated as[13]

$$\oint_{\partial V} G(\mathbf{r}_P,\mathbf{r})G^*(\mathbf{r}_Q,\mathbf{r})dS = -\frac{\rho}{2ik}(G(\mathbf{r}_P,\mathbf{r}_Q) - G^*(\mathbf{r}_P,\mathbf{r}_Q)), \quad (1)$$

where $\mathbf{r}_P$ and $\mathbf{r}_Q$ denote the locations of receivers. In this expression we assumed that the mass density ρ and wavenumber k are constant on the boundary ∂V, and the asterisk denotes complex conjugation. Throughout this paper we use a formulation in the frequency domain using the following Fourier convention: $F(t)=\int f(\omega)\exp(-i\omega t)d\omega$, with ω the angular frequency. For brevity we omit the frequency dependence in the remainder of this work.

For media with discrete scatterers or reflectors, the Green's function can be seen as a superposition of the waves that propagate along all possible scattering paths. Both Green's functions in the left hand side of Eq. (1) contain a sum over all scattering paths from the integration point $\mathbf{r}$ to the locations $\mathbf{r}_P$ and $\mathbf{r}_Q$, respectively. The left hand side of expression (1) therefore consists of a double sum over scattering paths that end at $\mathbf{r}_P$ and $\mathbf{r}_Q$, respectively. An example of two such paths is shown in Fig. 1. Let us denote the travel time for the path on the left as t_{S1AP} and the path on the right as t_{S2BQ}. In the time domain, the arrival time of the signal obtained by cross-correlation is given by the difference of the arrival times of the waves that are being cross-correlated. The cross-correlation of the waves that propagate along the paths of Fig. 1 thus produces a wave arriving at time $t_{S1AP} - t_{S2BQ}$. This travel time does not correspond to a physical wave that propagates between the points P and Q via the scattering path $A12B$. Such a contribution thus is a *spurious arrival* that does not correspond to a physical wave. These spurious arrivals arise because of the cross-correlation of wave propagating along different scattering paths, we refer to such contributions as *cross terms*. Expression (1) guarantees, though, that the left hand side gives the true Green's function after integration over surrounding sources, hence the spurious arrivals should disappear after integration over all sources. Earlier work treated the cancellation of spurious arrival in the case of one scatterer,[20] here we analyze the mechanism by which spurious arrivals cancel upon integration over sources in multiple scattering acoustic media with isolated scatterers.

Let us consider the cross-terms between different scattering paths in more detail. When we consider two different scattering paths that propagate from a source S to receivers P and Q, there are two possibilities; the first scatterer along these paths is the same (Fig. 2), or the first scatterer on both paths is different (Fig. 1). Suppose that there are M scatterers in the medium, then there are M ways in which one can choose the first scatterer in Fig. 2. In contrast, for the cross-terms along the paths shown in Fig. 1 the are $M(M-1)$ ways to choose the first scatterers along those paths. For a medium with many scatterers, the cross-terms in Fig. 1 are thus more prevalent than the cross-terms shown in Fig. 2. We show in

[a]Author to whom correspondence should be addressed. Electronic mail: rsnieder@mines.edu

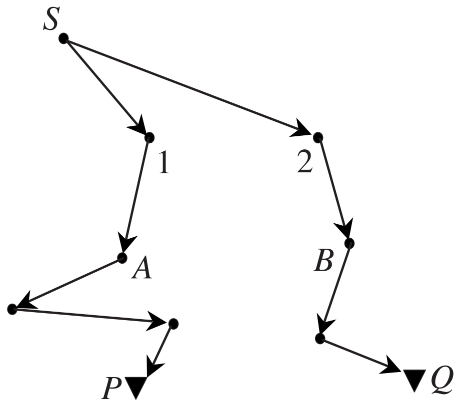

FIG. 1. Two scattering paths from a source S to receivers at points P and Q where the first scatterer along each path is different.

this work that despite the fact that that number of scattering paths shown in Fig. 1 is much larger than those in Fig. 2, it is the superposition of the scattering paths in both figures that leads to the cancellation of spurious arrivals.

We review the employed scattering theory in Section II. In Section III we show how the integrals that arise in the cross-correlation can be evaluated in the stationary phase approximation. In Section IV we derive the central result that the sum of most of these contributions vanishes by virtue of the generalized optical theorem. In Section V we evaluate the final nonzero contribution of the cross-correlation of waves that propagate from the source to scatterers to a common scattering path, and show that this correctly gives the scattered wave that propagates along that path. An essential element in the retrieval of scattered waves is that one needs cross-terms of the direct wave and scattered waves. In fact, when the Green's function retrieval is based on scattered waves only, the spurious arrivals do not vanish and one does *not* retrieve the scattered waves.

II. THE MULTIPLE SCATTERED WAVES

In this work we consider a homogeneous acoustic medium in which isolated scatterers are embedded. The employed acoustic wave equation is given by

$$\nabla \cdot \left(\frac{1}{\rho} \nabla p \right) + \frac{\omega^2}{\kappa} p = q, \tag{2}$$

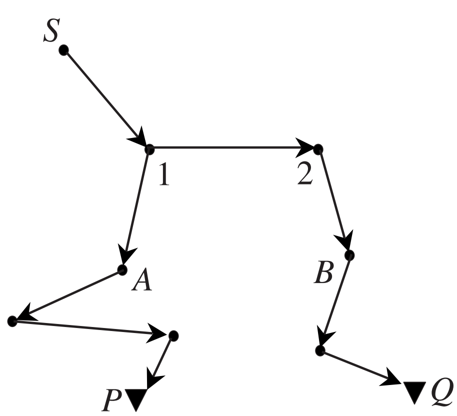

FIG. 2. Two scattering paths from a source S to receivers at points P and Q that share the first scatterer along the paths.

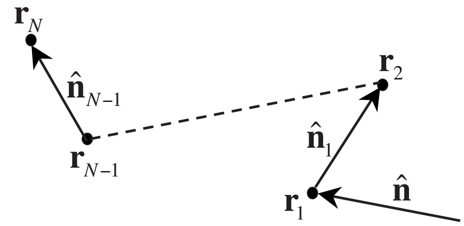

FIG. 3. Definition of geometric variables for a scattering path involving N scatterers.

where κ is the bulk modulus, and q the (injection) source. The Green's function $G(\mathbf{r},\mathbf{r}_0)$ is defined as the solution of expression (2) with $q(\mathbf{r})=\delta(\mathbf{r}-\mathbf{r}_0)$. The Green's function of the homogeneous reference medium in which the scatterers are embedded is

$$G_0(\mathbf{r},\mathbf{r}') = -\frac{\rho}{4\pi}\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}, \tag{3}$$

with the wavenumber given by $k=\omega\sqrt{\rho/\kappa}$. Scatterer j has scattering amplitude $f_j(\hat{\mathbf{n}},\hat{\mathbf{n}}')$,[21] where $\hat{\mathbf{n}}'$ is the direction of the incoming wave and $\hat{\mathbf{n}}$ the direction of the outgoing wave. The contribution to the Green's function of the wave propagating from a source at $\mathbf{r}_0$ via scatterers $1\cdots N$ at locations $\mathbf{r}_1,\cdots,\mathbf{r}_N$ to a receiver at $\mathbf{r}$ is given by

$$G^{path1\cdots N}(\mathbf{r},\mathbf{r}_0) = -\frac{\rho}{4\pi}\frac{e^{ik|\mathbf{r}-\mathbf{r}_N|}}{|\mathbf{r}-\mathbf{r}_N|}f_N(\hat{\mathbf{n}}_N,\hat{\mathbf{n}}_{N-1}) \times \frac{e^{ik|\mathbf{r}_N-\mathbf{r}_{N-1}|}}{|\mathbf{r}_N-\mathbf{r}_{N-1}|}\cdots f_1(\hat{\mathbf{n}}_1,\hat{\mathbf{n}}_0)\frac{e^{ik|\mathbf{r}_1-\mathbf{r}_0|}}{|\mathbf{r}_1-\mathbf{r}_0|}, \tag{4}$$

where the unit vector $\hat{\mathbf{n}}_i$ points from $\mathbf{r}_i$ to $\mathbf{r}_{i+1}$. In this expression, the propagation between scatterers i and j is denoted by $\exp(ik|\mathbf{r}_i-\mathbf{r}_j|)/|\mathbf{r}_i-\mathbf{r}_j|$. This description of scattering is valid when the scatterers are in each others far field. When this condition is not valid one can expand the scattering coefficients in a sum over spherical harmonics and replace the propagators by spherical Hankel functions;[22] in that case the analysis presented here is not applicable. The same scatterer can occur twice, or more, along the path, allowing for loops.

In the remainder of this work we focus on one particular scattering path, the treatment presented here is applicable to each scattering path separately. For brevity we introduce the following notation

$$G_{1\cdots N}(\hat{\mathbf{n}}) = \frac{e^{ik|\mathbf{r}_N-\mathbf{r}_{N-1}|}}{|\mathbf{r}_N-\mathbf{r}_{N-1}|}f_{N-1}(\hat{\mathbf{n}}_{N-2},\hat{\mathbf{n}}_{N-1}) \cdots \frac{e^{ik|\mathbf{r}_2-\mathbf{r}_1|}}{|\mathbf{r}_2-\mathbf{r}_1|}f_1(\hat{\mathbf{n}}_1,\hat{\mathbf{n}}), \tag{5}$$

where the corresponding scattering path and variables are defined in Fig. 3. This quantity describes the wave propagation for a wave incident from direction $\hat{\mathbf{n}}$ on scatterer 1, and then propagates via scatterers $1,2,\cdots,N-1$ to location $\mathbf{r}_N$. A comparison of expressions (4) and (5) shows that

$$G^{path1\cdots N}(\mathbf{r},\mathbf{r}_0) = -\frac{\rho}{4\pi}\frac{e^{ikr_{01}}}{r_{01}}G_{1\cdots N}(\hat{\mathbf{r}}_{01}). \tag{6}$$

Throughout this work we use the notations

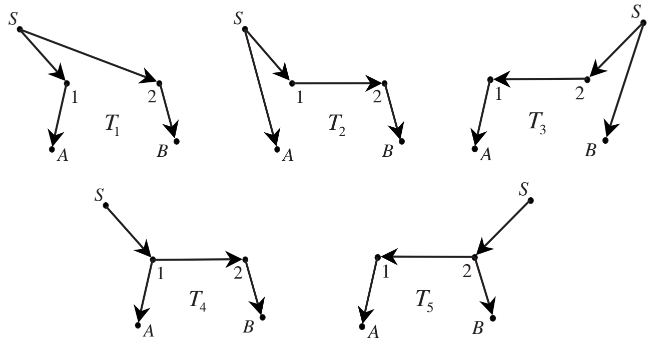

FIG. 4. Scattering diagrams for wave propagation from a source S to points A and B that visit the scatterers 1 and 2 one or two times. For simplicity the scattering paths from scatterer A to receiver P and scatterer B to receiver Q are not shown.

$$\mathbf{r}_{IJ}=\mathbf{r}_J-\mathbf{r}_I \text{ and } r_{IJ}=|\mathbf{r}_{IJ}|, \tag{7}$$

hence in expression (6), $\mathbf{r}_{01}=\mathbf{r}_1-\mathbf{r}_0$ is the vector pointing from a source at $\mathbf{r}_0$ to the position $\mathbf{r}_1$ of the first scatterer along the path considered.

III. SPURIOUS ARRIVALS FROM CROSS-TERMS

The waves traveling from the source S to receivers at $\mathbf{r}_P$ and $\mathbf{r}_Q$ either encounter different scatterers as the first scatterer along their paths, as shown in Fig. 1, or they may encounter the same first scatterer along their paths, see Fig. 2. In the notation of Fig. 1 we first consider scatterers "1" and "2." The next points along these paths are denoted with the labels "A" and "B." These points can either be scatterers, or the receivers where the wave field is recorded. The scatterers along the path considered are not necessarily spatially adjacent, the figures only show them in spatial order for reasons of clarity. The scattering paths beyond points A and B is independent of the location of the source, and in the following we don't show the continuation of those paths to the receivers P and Q.

We consider the scattering diagrams shown in Fig. 4. These diagrams show all the waves that propagate from the source and visit the scatterers 1 and 2 once or twice. As mentioned earlier, we do not show the fate of the waves beyond the points A and B because this part of the wave paths does not change during the integration over the sources on ∂V. There are five such diagrams, and in the following we compute the contribution of each diagram to the cross-correlation. We evaluate the contribution of each diagram using the stationary phase approximation[23,24] which becomes exact as the surface ∂V goes to infinity.[25] Note that the diagrams T_1 and T_2 are topologically identical in the sense that both diagrams describe a cross term between scattered waves that travel from the source to consecutive scatterers along the scattering path. Diagram T_2 follows from diagram T_1 by substituting $1\rightarrow A$ and $2\rightarrow 1$. This equivalence of diagrams is used in Section V where we sum over all scatterers along the scattering path that we consider.

We first analyze the term T_1 that corresponds to the diagram in the top left of Fig. 4. Using expressions (4) and (5), the wave that propagates along the left path of term T_1 in Fig. 4 from the source S via the scatterer A a receiver P is given by

$$u_{left}=-\frac{\rho}{4\pi}\frac{e^{ikr_{S1}}}{r_{S1}}f_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}}_{S1})\frac{e^{ikr_{1A}}}{r_{1A}}G_{A\cdots P}(\hat{\mathbf{r}}_{1A}), \tag{8}$$

where $G_{A\cdots P}(\hat{\mathbf{r}}_{1A})$ accounts for the propagation from scatterer A to receiver P along the scattering path. The subscript S refers to the source location. The unit vector $\hat{\mathbf{r}}_{1A}$ is defined using expression (7). Similarly, the wave propagating along the right path of term T_1 in Fig. 4 is given by

$$u_{right}=-\frac{\rho}{4\pi}\frac{e^{ikr_{S2}}}{r_{S2}}f_2(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{S2})\frac{e^{ikr_{2B}}}{r_{2B}}G_{B\cdots Q}(\hat{\mathbf{r}}_{2B}). \tag{9}$$

The contribution to the cross-correlation of these two paths is given by

$$T_1=\oint\left(-\frac{\rho}{4\pi}\frac{e^{ikr_{1A}}}{r_{1A}}f_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}}_{S1})\frac{e^{ikr_{S1}}}{r_{S1}}G_{A\cdots P}(\hat{\mathbf{r}}_{1A})\right)\times\left(-\frac{\rho}{4\pi}\frac{e^{ikr_{2B}}}{r_{2B}}f_2(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{S2})\frac{e^{ikr_{S2}}}{r_{S2}}G_{B\cdots Q}(\hat{\mathbf{r}}_{2B})\right)^{*}dS, \tag{10}$$

where the integration is over sources on a spherical surface surrounding the scatterers and the receivers. Rearranging terms, T_1 can be written as

$$T_1=\left(\frac{\rho}{4\pi}\right)^2\frac{e^{ik(r_{1A}-r_{2B})}}{r_{1A}r_{2B}}G_{A\cdots P}(\hat{\mathbf{r}}_{1A})G^{*}_{B\cdots Q}(\hat{\mathbf{r}}_{2B})\times\oint\frac{e^{ik(r_{S1}-r_{S2})}}{r_{S1}r_{S2}}f_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}}_{S1})f_2^{*}(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{S2})dS. \tag{11}$$

The surface integral can be evaluated with the stationary phase approximation following the steps taken in Refs. 20 and 26. Instead of repeating these steps, we recognize that, apart from the terms containing the scattering amplitude, the surface integral is equal to the superposition of the causal and a-causal unperturbed Green's function of Equation (3):

$$\left(\frac{\rho}{4\pi}\right)^2\oint\frac{e^{ik(r_{S1}-r_{S2})}}{r_{S1}r_{S2}}dS=\oint G_0(\mathbf{r}_1,\mathbf{r})G_0^{*}(\mathbf{r}_2,\mathbf{r})dS$$
$$=-\frac{\rho}{2ik}(G_0(\mathbf{r}_1,\mathbf{r}_2)-G_0^{*}(\mathbf{r}_1,\mathbf{r}_2))$$
$$=\frac{\rho^2}{8\pi ik}\left(\frac{e^{ikr_{12}}}{r_{12}}-\frac{e^{-ikr_{12}}}{r_{12}}\right), \tag{12}$$

where the first and last identities follow from Eq. (3) and the second equality from expression (1). We use this result in the stationary phase approximation of the integral (11), but must insert the stationary phase locations for the source position in the variables that depend on the source position.

Following the analysis of Refs. 20 and 26, the surface integral in Eq. (11) has two stationary phase points that are shown in Fig. 5. For the stationary phase point in the left panel of Fig. 5, $\hat{\mathbf{r}}_{S1}=\hat{\mathbf{r}}_{S2}=\hat{\mathbf{r}}_{12}$, and $r_{S1}-r_{S2}=-r_{12}$. For the stationary phase point of the right panel $\hat{\mathbf{r}}_{S1}=\hat{\mathbf{r}}_{S2}=-\hat{\mathbf{r}}_{12}$, and $r_{S1}-r_{S2}=r_{12}$. Using these results, expression (11) is in the stationary phase approximation given by

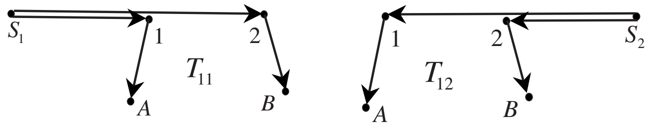

FIG. 5. Stationary points for the source integration in expression (11) for term T_1.

$$T_1 = \frac{\rho^2}{8\pi ik}\frac{e^{ik(r_{1A}-r_{2B})}}{r_{1A}r_{2B}}\left(\frac{e^{ikr_{12}}}{r_{12}}f_1(\hat{\mathbf{r}}_{1A},-\hat{\mathbf{r}}_{12})f_2^*(\hat{\mathbf{r}}_{2B},-\hat{\mathbf{r}}_{12}) - \frac{e^{-ikr_{12}}}{r_{12}}f_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}}_{12})f_2^*(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{12})\right) \times G_{A\cdots P}(\hat{\mathbf{r}}_{1A})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{2B}). \tag{13}$$

Note that apart from contributions from the scattering amplitude, the phase of the first term in this expression is given by $k(r_{1A}+r_{12}-r_{2B})$. In the time domain this corresponds to a wave arriving at time $t=(r_{1A}+r_{12}-r_{2B})/c$, where c is the wave velocity of the reference medium. This wave, with an arrival time that depends on the *difference* of the travel times $(r_{A1}+r_{12})/c$ and r_{2B}/c along the scattering path rather than its *sum*, does not correspond to a physical arrival. The same consideration holds for the second term in expression (13), whose phase depends on $k(r_{1A}-r_{12}-r_{2B})$. Hence both terms of T_1 are spurious arrivals that must ultimately be cancelled by other terms.

IV. CANCELLATION OF THE SPURIOUS ARRIVALS

In this section we analyze the contributions of the diagrams T_2 through T_5 shown in Fig. 4. Using expression (4), the term T_2 can be written as

$$T_2 = \oint\left(-\frac{\rho}{4\pi}\frac{e^{ikr_{SA}}}{r_{SA}}G_{A\cdots P}(\hat{\mathbf{r}}_{SA})\right)\left(-\frac{\rho}{4\pi}\frac{e^{ikr_{2B}}}{r_{2B}}f_2(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{12})\frac{e^{ikr_{12}}}{r_{12}}f_1(\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{S1})\frac{e^{ikr_{S1}}}{r_{S1}}G_{B\cdots Q}(\hat{\mathbf{r}}_{2B})\right)^* dS$$
$$= \left(\frac{\rho}{4\pi}\right)^2\frac{e^{-ikr_{2B}}}{r_{2B}}\frac{e^{-ikr_{12}}}{r_{12}}f_2^*(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{12})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{2B})\oint\frac{e^{ik(r_{SA}-r_{S1})}}{r_{2A}r_{S1}}G_{A\cdots P}(\hat{\mathbf{r}}_{SA})f_1^*(\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{S1})dS. \tag{14}$$

The integral can be evaluated in the stationary phase approximation We consider the contribution of the stationary phase using expression (12) and evaluate the scattering amplitude for incoming waves excited at each stationary source position. The stationary phase points are shown in Fig. 6, and their contribution is given by

$$T_2 = T_{21} + T_{22}, \tag{15}$$

with

$$T_{21} = \frac{\rho^2}{8\pi ik}\frac{e^{ik(r_{1A}-r_{2B})}}{r_{1A}r_{2B}}\frac{e^{-ikr_{12}}}{r_{12}}f_1^*(\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{1A}) \times f_2^*(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{12})G_{A\cdots P}(\hat{\mathbf{r}}_{1A})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{2B}), \tag{16}$$

and

$$T_{22} = -\frac{\rho^2}{8\pi ik}\frac{e^{-ik(r_{1A}+r_{2B})}}{r_{1A}r_{2B}}\frac{e^{-ikr_{12}}}{r_{12}}f_1^*(\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{A1}) \times f_2^*(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{12})G_{A\cdots P}(\hat{\mathbf{r}}_{A1})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{2B}), \tag{17}$$

where we used $\hat{\mathbf{r}}_{S1}=\hat{\mathbf{r}}_{A1}$ for the stationary phase point in the left panel of Fig. 6, and $\hat{\mathbf{r}}_{S1}=\hat{\mathbf{r}}_{1A}$ for the other stationary point. Note that the directions $1A$ and $A1$ are reversed in expressions (16) and (17) because of the opposite orientation of the stationary phase points in Fig. 6. In the time domain term T_{21} corresponds to a wave arriving at time $t=(r_{1A}-r_{2B}-r_{12})/c$. Because it contains the difference of arrival times, it does not correspond to any physical wave that propagates between the scatterers.

Term T_3 can be obtained from the analysis for T_2 by interchanging points A and B, points 1 and 2 and taking the complex conjugate. Applying these substitutions to expression (16) gives for the spurious arrival of T_{31} due to one of the stationary phase points

$$T_{31} = -\frac{\rho^2}{8\pi ik}\frac{e^{ik(r_{1A}-r_{2B})}}{r_{1A}r_{2B}}\frac{e^{ikr_{12}}}{r_{12}}f_1(\hat{\mathbf{r}}_{1A},-\hat{\mathbf{r}}_{12})f_2(-\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{2B}) \times G_{A\cdots P}(\hat{\mathbf{r}}_{1A})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{2B}), \tag{18}$$

where we used that $\hat{\mathbf{r}}_{21}=-\hat{\mathbf{r}}_{12}$. This is, again, a spurious arrival because it corresponds to a wave arriving at a time difference $t=(r_{1A}-r_{2B}+r_{12})/c$. The contribution from the other stationary phase point follows by making the substitutions given above in expression (17) and is given by

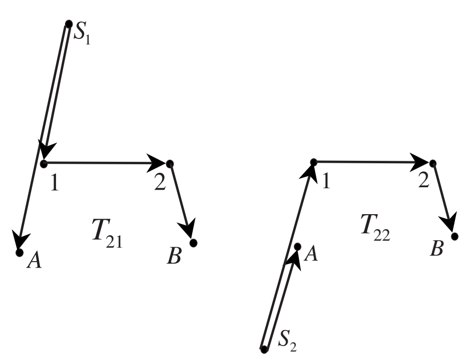

FIG. 6. Stationary points for the source integration in expression (14) for term T_2.

$$T_{32} = \frac{\rho^2}{8\pi ik}\frac{e^{ik(r_{1A}+r_{2B})}}{r_{1A}r_{2B}}\frac{e^{ikr_{12}}}{r_{12}}f_1(\hat{\mathbf{r}}_{1A},-\hat{\mathbf{r}}_{12})f_2(-\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{B2})G_{A\cdots P}(\hat{\mathbf{n}}_{1A})G^*_{B\cdots Q}(\hat{\mathbf{n}}_{B2}). \tag{19}$$

Using Eq. (4) the contributions to term T_4 of the paths shown in Fig. 4 are given by

$$\begin{aligned} T_4 &= \oint\left(-\frac{\rho}{4\pi}\frac{e^{ikr_{1A}}}{r_{1A}}f_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}}_{S1})\frac{e^{ikr_{S1}}}{r_{S1}}G_{A\cdots P}(\hat{\mathbf{r}}_{1A})\right) \\ &\quad\times\left(-\frac{\rho}{4\pi}\frac{e^{ikr_{2B}}}{r_{2B}}f_2(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{12})\frac{e^{ikr_{12}}}{r_{12}}f_1(\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{S1})\frac{e^{ikr_{S1}}}{r_{S1}}G_{B\cdots Q}(\hat{\mathbf{r}}_{2B})\right)^* dS \\ &= \frac{\rho^2}{16\pi^2}\frac{e^{ik(r_{1A}-r_{2B})}}{r_{1A}r_{2B}}\frac{e^{-ikr_{12}}}{r_{12}}f_2^*(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{12})\oint\frac{1}{r_{S1}^2}f_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}}_{S1})f_1^*(\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{S1})dS\times G_{A\cdots P}(\hat{\mathbf{r}}_{A1})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{2B}). \end{aligned} \tag{20}$$

The surface element dS is related to the increment $d\Omega$ in solid angle by the relation $(1/r_{S1}^2)dS = d\Omega$. Replacing $\hat{\mathbf{r}}_{S1}$, which depends on the source position over which we integrate, by a new integration variable $\hat{\mathbf{r}}$ gives

$$\begin{aligned} T_4 &= \frac{\rho^2}{16\pi^2}\frac{e^{ik(r_{1A}-r_{2B})}}{r_{1A}r_{2B}}\frac{e^{-ikr_{12}}}{r_{12}}f_2^*(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{12}) \\ &\quad\times\left(\oint f_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}})f_1^*(\hat{\mathbf{r}}_{12},\hat{\mathbf{r}})d\Omega\right)G_{A\cdots P}(\hat{\mathbf{r}}_{A1})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{2B}). \end{aligned} \tag{21}$$

Term T_5 of Fig. 4 follows from this expression by interchanging $A\leftrightarrow B$, $1\leftrightarrow 2$, and taking the complex conjugate

$$\begin{aligned} T_5 &= \frac{\rho^2}{16\pi^2}\frac{e^{ik(r_{1A}-r_{2B})}}{r_{1A}r_{2B}}\frac{e^{ikr_{12}}}{r_{12}}f_1(\hat{\mathbf{r}}_{1A},-\hat{\mathbf{r}}_{12}) \\ &\quad\times\left(\oint f_2^*(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}})f_2(-\hat{\mathbf{r}}_{12},\hat{\mathbf{r}})d\Omega\right)G_{A\cdots P}(\hat{\mathbf{r}}_{A1})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{2B}). \end{aligned} \tag{22}$$

Note that T_4 and T_5 also depend on the difference of path lengths, and thus are unphysical arrivals.

The sum $T_{spur} = T_1 + T_{21} + T_{31} + T_4 + T_5$ of the spurious terms of Eqs. (13), (16), (18), (21), and (22) gives after a rearrangement of terms

$$\begin{aligned} T_{spur} &= \frac{\rho^2 e^{ik(r_{1A}-r_{2B})}}{4\pi k r_{1A}r_{2B}}\left(\frac{e^{ikr_{12}}}{r_{12}}f_1(\hat{\mathbf{r}}_{1A},-\hat{\mathbf{r}}_{12})F_2^*(\hat{\mathbf{r}}_{2B},-\hat{\mathbf{r}}_{12})\right. \\ &\quad\left.+\frac{e^{-ikr_{12}}}{r_{12}}f_2^*(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{12})F_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}}_{12})\right) \\ &\quad\times G_{A\cdots P}(\hat{\mathbf{r}}_{A1})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{2B}) \end{aligned} \tag{23}$$

with

$$\begin{aligned} F_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}}_{12}) &= -\frac{1}{2i}f_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}}_{12}) + \frac{1}{2i}f_1^*(\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{1A}) \\ &\quad+\frac{k}{4\pi}\oint f_1(\hat{\mathbf{r}}_{1A},\hat{\mathbf{r}})f_1^*(\hat{\mathbf{r}}_{12},\hat{\mathbf{r}})d\Omega, \end{aligned} \tag{24}$$

and

$$\begin{aligned} F_2^*(\hat{\mathbf{r}}_{2B},-\hat{\mathbf{r}}_{12}) &= \frac{1}{2i}f_2^*(\hat{\mathbf{r}}_{2B},-\hat{\mathbf{r}}_{12}) - \frac{1}{2i}f_2(-\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{2B}) \\ &\quad+\frac{k}{4\pi}\oint f_2^*(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}})f_2(-\hat{\mathbf{r}}_{12},\hat{\mathbf{r}})d\Omega. \end{aligned} \tag{25}$$

The scatterers 1 and 2 must both satisfy the generalized optical theorem

$$\frac{1}{2i}(f_j(\hat{\mathbf{r}}_A,\hat{\mathbf{r}}_B) - f_j^*(\hat{\mathbf{r}}_B,\hat{\mathbf{r}}_A)) = \frac{k}{4\pi}\oint f_j(\hat{\mathbf{r}}_A,\hat{\mathbf{r}})f_j^*(\hat{\mathbf{r}}_B,\hat{\mathbf{r}})d\Omega. \tag{26}$$

This theorem has been derived for quantum mechanics[27,28] and acoustics.[29] By virtue of this theorem, both F_1 and F_2 in expressions (24) and (25) vanish. Because of Eq. (23) the sum T_{spur} of the spurious arrivals of the diagrams of Fig. 4 is thus equal to zero:

$$T_1 + T_{21} + T_{31} + T_4 + T_5 = 0. \tag{27}$$

Since this sum vanishes, the only nonzero contribution of the sum of the terms T_1 through T_5 comes from the terms T_{22} and T_{32}, hence the sum $T = T_1 + T_2 + T_3 + T_4 + T_5$ of all diagrams in Fig. 4 is given by

$$\begin{aligned} T &= \frac{\rho^2}{8\pi ik}\frac{e^{ik(r_{1A}+r_{12}+r_{2B})}}{r_{1A}r_{12}r_{2B}}f_1(\hat{\mathbf{r}}_{1A},-\hat{\mathbf{r}}_{12}) \\ &\quad\times f_2(-\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{B2})G_{A\cdots P}(\hat{\mathbf{r}}_{1A})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{B2}) \\ &\quad-\frac{\rho^2}{8\pi ik}\frac{e^{-ik(r_{1A}+r_{12}+r_{2B})}}{r_{1A}r_{12}r_{2B}}f_1^*(\hat{\mathbf{r}}_{12},\hat{\mathbf{r}}_{A1})f_2^*(\hat{\mathbf{r}}_{2B},\hat{\mathbf{r}}_{12}) \\ &\quad\times G_{A\cdots P}(\hat{\mathbf{r}}_{A1})G^*_{B\cdots Q}(\hat{\mathbf{r}}_{2B}). \end{aligned} \tag{28}$$

In the next section we consider the sum of the diagrams T_1 through T_5 and their contribution to expression (28) for all scatterers along the scattering path, and we treat the sum of all those contributions.

V. SUMMING THE CONTRIBUTIONS ALONG THE SCATTERING PATH

According to Eq. (28) the total contribution of the diagrams T_1 through T_5 of Fig. 4 reduces to the term T_{22} of Fig. 6 and a corresponding diagram T_{32} that connects to scatterer

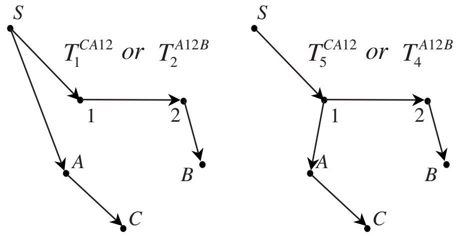

FIG. 7. Two different ways of accounting for the cross-terms of two scattering paths.

CA	A	A1	1	12	2	2B	⋯
				$\frac{1}{2}(T_{22}+T_{21})$	$\frac{1}{2}T_4$	$0\cdot(T_{11}+T_{12})$	⋯
		$\frac{1}{2}(T_{22}+T_{21})$	$\frac{1}{2}T_4$	$0\cdot(T_{11}+T_{12})$	$\frac{1}{2}T_5$	$\frac{1}{2}(T_{31}+T_{32})$	
$\frac{1}{2}(T_{22}+T_{21})$	$\frac{1}{2}T_4$	$0\cdot(T_{11}+T_{12})$	$\frac{1}{2}T_5$	$\frac{1}{2}(T_{31}+T_{32})$		$=0$	
					$=0$		

FIG. 9. Cancellation of terms $T_1+T_{21}+T_{31}+T_4+T_5$ for adjacent groups of scatterers along the path. Rows and columns have the same meaning as in Fig. 8. The sum of all terms in each dashed box cancels when the contributions from rows above and below indicated by solid arrows are included.

B. We show later in this section that these remaining terms gives the superposition of scattered waves that propagate in opposite direction along the path.

Consider waves propagate that from a scatterer A to another point C, which can either be the next scatterer along the path or a receiver, as shown in Fig. 7. There is an ambiguity how we label the different terms for adjacent groups of four points. Consider first the right panel of Fig. 7. Comparing it with Fig. 4 it can be considered to be either term T_4 for the points $A12B$ or term T_5 for the points $CA12$, because in both interpretations the waves that are cross-correlated propagate from the source to scatterer 1 and then continue in opposite ways along the scattering path. Similarly, a comparison with Fig. 4 shows that the cross-terms in the left panel of Fig. 7 can be interpreted either as term T_2 for the points $A12B$ or as term T_1 for points $CA12$. A similar ambiguity exists between terms T_3 and T_1 when continuing the scattering path beyond scatterer B in Fig. 7.

One must take care not to count ambiguous terms twice when summing over all cross-terms of scattered waves along the scattering path. This can be avoided by assigning weights as shown in Fig. 8 to the different cross-terms. The entries in the figure should be interpreted as follows. The term T_4 in the column "1" denotes, for example, the diagram shown in the right panel of Fig. 7. It is entered in the middle column because scatterer 1 is the first scatterer encountered after leaving the source. The entry is shown in the middle row, because this row denotes terms for the points $A12B$. We know from the right panel of Fig. 7 that this term equals term T_5 for the points $CA12$, hence term T_5 is also indicated in column marked "1" in the bottom row that is applicable to the points $CA12$. By giving each of these terms a weight $\frac{1}{2}$, as shown in Fig. 8, we avoid counting these terms twice.

CA	A	A1	1	12	2	2B	⋯
				$\frac{1}{2}T_2$	$\frac{1}{2}T_4$	$0\cdot T_1$	⋯
		$\frac{1}{2}T_2$	$\frac{1}{2}T_4$	$0\cdot T_1$	$\frac{1}{2}T_5$	$\frac{1}{2}T_3$	
$\frac{1}{2}T_2$	$\frac{1}{2}T_4$	$0\cdot T_1$	$\frac{1}{2}T_5$	$\frac{1}{2}T_3$			

FIG. 8. Weights given to terms T_1 through T_5 for adjacent groups of scatterers along the path to ensure that each diagram is counted once. Column "1" denotes, for example, cross-terms of scattered waves that both meet scatterer 1 as first scatterer, while column "$A1$" denotes cross-terms of waves that encounter scatterers A and 1, respectively, as first scatterer along their paths. The bottom row denotes the contribution for points $CA12$, the middle row the contribution of points $A12B$, etc. There are rows above and below for other sets of four scattering points along the scattering paths.

Similarly, according to the left panel of Fig. 7 we should avoid double counting term T_2 for the points $A12B$ and term T_1 for the points $CA12$, because these terms denote the same cross-term of scattered waves. Term T_2 is indicated in the column "$A1$" and the middle row of Fig. 8, because, as shown in Fig. 7, this term is a cross term of waves with scatterers A and 1 as the first scatterers encountered along the paths considered. This term is also entered as a T_1-contribution to the bottom row in the column "$A1$." We assign weights to these terms so that when one sums along all groups of four scatterers along the path the total contribution of these terms adds up to a weight 1. This can be seen, for example in the column "12" of Fig. 8. It may seem surprising that the term T_1 is assigned a zero weight, but, as we show later, the contribution of this term is covered by the terms T_2 and T_3 of adjacent groups of four points.

Next we sum the contributions of each adjacent group of four points. This corresponds to a sum over the rows in Fig. 8 or Fig. 9. We first compute the sum over the elements in the middle row of Fig. 9, it is equal to

$$\begin{aligned}
&\tfrac{1}{2}T_2^{A12B}+\tfrac{1}{2}T_4^{A12B}+0\cdot T_1^{A12B}+\tfrac{1}{2}T_5^{A12B}+\tfrac{1}{2}T_3^{A12B}\\
&\quad=\tfrac{1}{2}(T_{21}^{A12B}+T_{22}^{A12B}+T_1^{A12B}+T_5^{A12B}+T_{31}^{A12B}+T_{32}^{A12B})\\
&\quad-\tfrac{1}{2}T_1^{A12B}\\
&\quad=\tfrac{1}{2}T_{22}^{A12B}+\tfrac{1}{2}T_{32}^{A12B}-\tfrac{1}{2}T_{11}^{A12B}-\tfrac{1}{2}T_{12}^{A12B},
\end{aligned} \tag{29}$$

where we added and subtracted $\frac{1}{2}T_1^{A12B}$ in the first identity, and used expression (27) in the second identity. We also used Eq. (15) that expresses the contribution of terms T_1 through T_3 into the two stationary point contributions. Expression (29) holds for each row. Applying this expression to the bottom row of Fig. 9 gives a contribution $\frac{1}{2}T_{32}^{CA12}$ which is the opposite of the remaining term $-\frac{1}{2}T_{12}^{A12B}$ in Eq. (29) for the middle row. (This equivalence of these diagrams can be verified by drawing a diagram similar to that in the left panel of Fig. 7.) These terms thus cancel, this is indicated in Fig. 9 by the solid arrow pointing upward in column "12." Using the same reasoning, the remaining term $\frac{1}{2}T_{22}$ of the top row cancels the remaining contribution $-\frac{1}{2}T_{11}^{A12B}$ in Eq. (29) for the middle row, this is indicated by the solid arrow pointing downward in column "12" of Fig. 9. The terms $\frac{1}{2}T_{22}^{A12B}$ $+\frac{1}{2}T_{32}^{A12B}$ in expression (29) for the middle row cancel T_1-contributions in the rows below and above, respectively. This cancellation is indicated by solid arrows in Fig. 9. Ultimately all contributions of terms in Fig. 9 cancel, with the

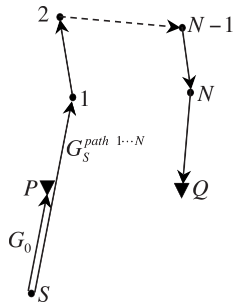

FIG. 10. The stationary source position that gives a nonzero contribution the scattering path $P1\cdots NQ$.

exception of remaining terms at the end points on the scattering path.

In the following we evaluate the contribution of Eq. (28) at the end points of the scattering path. As argued above, the contributions of expression (28) that end at the receivers P or Q are the only terms that give a nonzero contribution. For ease of notation, we rename the scatterers along the path with indices $1,2,\ldots,N$; this index enumerates the scatterers along the path starting at receiver P. We first consider the cross-term that remains at the end of scattering path at receiver P as shown in Fig. 10. This cross-term consists of the direct wave G_0 that travels to receiver P with the scattered wave $G_S^{path\ 1\cdots N}$ that propagates along the scatterers $1,\ldots,N$ to receiver Q.

The contribution of this cross-term is, in the notation of Fig. 10, given by

$$T_{22} = \oint \left(-\frac{\rho}{4\pi}\frac{e^{ikr_{SP}}}{r_{SP}}\right)\left(-\frac{\rho}{4\pi}\frac{e^{ikr_{S1}}}{r_{S1}}G_{1\cdots NQ}(\hat{\mathbf{r}}_{S1})\right)^* dS = \left(-\frac{\rho}{4\pi}\right)^2 \oint \frac{e^{ik(r_{SP}-r_{S1})}}{r_{SP}r_{S1}} G^*_{1\cdots NQ}(\hat{\mathbf{r}}_{S1}) dS. \quad (30)$$

Since we only need to account for the equivalent of term T_{22} we consider the contribution of the stationary phase point shown in Fig. 10, and using Eq. (12), the contribution of this stationary phase point is given by

$$T_{22} = -\frac{\rho^2}{8\pi ik}\frac{e^{-ikr_{P1}}}{r_{P1}}G^*_{1\cdots NQ}(\hat{\mathbf{r}}_{S1}) = -\frac{\rho^2}{8\pi ik}\left(-\frac{4\pi}{\rho}\right)(G^{path\ 1\cdots N}(\mathbf{r}_P,\mathbf{r}_Q))^* = \frac{\rho}{2ik}(G^{path\ 1\cdots N}(\mathbf{r}_P,\mathbf{r}_Q))^*, \quad (31)$$

where we used that at the stationary point $\hat{\mathbf{r}}_{S1}=\hat{\mathbf{r}}_{P1}$, and expression (6) in the second identity. As indicated in Fig. 10, this contribution consists of the correlation of the direct wave $G_0(\mathbf{r}_P,\mathbf{r}_S)$, that propagates from the source to the receiver at $\mathbf{r}_P$ with the scattered wave $G_S^{path\ 1\cdots N}(\mathbf{r}_Q,\mathbf{r}_S)$, that travels from the source via scatterers $1\cdots N$ to the receiver at $\mathbf{r}_Q$. The contribution from the term T_{32} at the other end of the scattering path follows by taking the complex conjugate, replacing P and Q, and reversing the order of the scatterers $(1\cdots N \to N\cdots 1)$, which gives

$$T_{32} = -\frac{\rho}{2ik}G^{path\ N\cdots 1}(\mathbf{r}_Q,\mathbf{r}_P) = -\frac{\rho}{2ik}G^{path\ 1\cdots N}(\mathbf{r}_P,\mathbf{r}_Q), \quad (32)$$

where we used reciprocity in the last identity. Adding the contributions from Eqs. (31) and (32) finally yields

$$T_{22}+T_{32} = -\frac{\rho}{2ik}(G^{path\ 1\cdots N}(\mathbf{r}_P,\mathbf{r}_Q) - (G^{path\ 1\cdots N}(\mathbf{r}_P,\mathbf{r}_Q))^*). \quad (33)$$

This is nothing but expression (1) for the wave propagating along the scattering path under consideration.

VI. DISCUSSION

We have shown for a multiple scattering system with discrete scatterers that the cross-correlation of different scattering paths vanishes when one integrates over all sources on a surface that bound the region with scatterers and receivers. One might think that the cancellation of spurious arrivals occurs because the phase of each of these arrivals is different for different pairs of scattering paths which results in destructive interference of spurious arrivals, but this is not the reason. The cancellation process involves the sum of the five scattering diagrams shown in Fig. 4, and the sum of scattering diagrams vanishes because every scatterer must satisfy the generalized optical theorem. The cancellation of spurious arrivals for multiple scattered waves shown here complements an earlier proof that for an isolated scatterer the spurious arrivals cancel.[20] Because of the extremely large number of spurious cross-terms in a multiple scattering medium, the cancellation of spurious arrivals is much more important in a multiple scattering medium than in a medium with just one scatterer. For weakly scattering media where scattering can be treated in the Born approximation, the cross-terms of scattered waves with scattered waves is of higher order and can thus be ignored in Green's function extraction.[30,31]

It is essential in the cancellation of the spurious arrivals that the power spectrum of the sources on the boundary ∂V is constant and that sources are present everywhere on this boundary because these requirements ensure that the surface integral in the Green's function extraction is adequately sampled. If these conditions are not met, the angular integrals in the terms T_1 through T_5 are multiplied with variations in the power spectrum and/or spatial density of sources, and as a result the spurious arrivals may not cancel.[20,32] This is important for practical reasons, since in applications there may be gaps in the source distribution on ∂V, and even if sources are present everywhere on ∂V, the power spectrum of these source may vary. In that case the spurious arrivals may contaminate estimates for the Green's function obtained from cross-correlation of field fluctuations.

As shown in Section V, the extraction of the wave propagating along the scattering path considered follows from the cross-correlation of the direct wave propagating to one receiver with the scattered wave propagating along the scattering path to the other receiver, because the cross-terms between scattered waves ultimately cancel. Suppose one estimates the Green's function by cross-correlating scattered waves only. In that case the cross-terms of scattered waves with the direct wave is missing and the extracted Green's function contains spurious arrivals. It has been noted earlier that the cross-correlation of scattered waves with scattered waves does *not* give the scattered waves,[20,33] and this study confirms that conclusion for multiple scattering media. The failure to extract scattered waves by cross-correlating only scattered waves is ultimately due to the fact that the scattered waves do not satisfy the wave equation.[34]

ACKNOWLEDGMENTS

We thank Filippo Broggini and an anonymous reviewer for their insightful and critical comments. This work was supported by the NSF through Grant EAS-0609595.

[1]E. Larose, L. Margerin, A. Derode, B. van Tiggelen, M. Campillo, N. Shapiro, A. Paul, L. Stehly, and M. Tanter, "Correlation of random wavefields: An interdisciplinary review," Geophysics **71**, SI11–SI21 (2006).

[2]A. Curtis, P. Gerstoft, H. Sato, R. Snieder, and K. Wapenaar, "Seismic interferometry—Turning noise into signal," The Leading Edge **25**, 1082–1092 (2006).

[3]*Seismic Interferometry: History and Present Status*, SEG Geophysics Reprints Series No. 26, edited by K. Wapenaar, D. Draganov, and J. O. A. Robertsson (Society of Exploration Geophysics, Tulsa, OK, 2008), pp. 1–628.

[4]G. Schuster, *Seismic Interferometry* (Cambridge University Press, Cambridge, UK, 2009), pp. 1–29.

[5]R. Snieder, M. Miyazawa, E. Slob, I. Vasconcelos, and K. Wapenaar, "A comparison of strategies for seismic interferometry," Surv. Geophys. **30**, 503–523 (2009).

[6]K. Wapenaar, D. Draganov, R. Snieder, X. Campman, and A. Verdel, "Tutorial on seismic interferometry. Part I: Basic principles and applications," Geophysics In press.

[7]K. Wapenaar, E. Slob, R. Snieder, and A. Curtis, "Tutorial on seismic interferometry. Part II: Underlying theory," Geophysics In press.

[8]R. F. Greene and H. B. Callen, "On the formulation of thermodynamic fluctuation theory," Phys. Rev. **83**, 1231–1235 (1951).

[9]H. B. Callen and T. A. Welton, "Irreversibility and generalized noise," Phys. Rev. **83**, 34–40 (1951).

[10]H. Nyquist, "Thermal agitation of electric charge in conductors," Phys. Rev. **32**, 110–113 (1928).

[11]S. M. Rytov, Y. A. Kravtsov, and V. I. Tatarskii, *Principles of Statistical Radiophysics. Vol. 3: Elements of Random Fields* (Springer, Berlin, 1989), pp. 146–148.

[12]K. Wapenaar, E. Slob, and R. Snieder, "Unified Green's function retrieval by cross-correlation," Phys. Rev. Lett. **97**, 234301 (2006).

[13]R. Snieder, K. Wapenaar, and U. Wegler, "Unified Green's function retrieval by cross-correlation; connection with energy principles," Phys. Rev. E **75**, 036103 (2007).

[14]R. L. Weaver, "Ward identities and the retrieval of Green's functions in the correlations of a diffuse field," Wave Motion **45**, 596–604 (2008).

[15]P. Gouédard, L. Stehly, F. Brenguier, M. Campillo, Y. Colin de Verdière, E. Larose, L. Margerin, P. Roux, F. J. Sánchez-Sesma, N. M. Shapiro, and R. L. Weaver, "Cross-correlation of random fields: Mathematical approach and applications," Geophys. Prospect. **56**, 375–393 (2008).

[16]E. Slob, R. Snieder, and A. Revil, "Retrieving electric resistivity data from self-potential measurements by cross-correlation," Geophys. Res. Lett. **37**, L04308 (2010).

[17]R. Snieder, E. Slob, and K. Wapenaar, "Lagrangian Green's function extraction, with applications to potential fields, diffusion, and acoustic waves," New J. Phys. **12**, 063013 (2010).

[18]A. Derode, E. Larose, M. Campillo, and M. Fink, "How to estimate the Green's function for a heterogeneous medium between two passive sensors? Application to acoustic waves," Appl. Phys. Lett. **83**, 3054–3056 (2003).

[19]K. Wapenaar, J. Fokkema, and R. Snieder, "Retrieving the Green's function by cross-correlation: A comparison of approaches," J. Acoust. Soc. Am. **118**, 2783–2786 (2005).

[20]R. Snieder, K. van Wijk, M. Haney, and R. Calvert, "The cancellation of spurious arrivals in Green's function extraction and the generalized optical theorem," Phys. Rev. E **78**, 036606 (2008).

[21]E. Merzbacher, *Quantum Mechanics*, 2nd ed. (Wiley, New York, 1970), pp. 222–227.

[22]P. A. Martin, *Multiple Scattering*, Encyclopedia of Mathematics and Its Applications Vol. **107** (Cambridge University Press, Cambridge, UK, 2006), pp. 132–134.

[23]N. Bleistein and R. A. Handelsman, *Asymptotic Expansions of Integrals* (Dover, New York, 1975), pp. 219–224.

[24]R. Snieder, *A Guided Tour of Mathematical Methods for the Physical Sciences*, 2nd ed. (Cambridge University Press, Cambridge, UK, 2004), pp. 450–460.

[25]H. C. Van De Hulst, "On the attenuation of plane waves by obstacles of arbitrary size and form," Physica (Amsterdam) **15**, 740–746 (1949).

[26]O. A. Godin, "Recovering the acoustic Green's function from ambient noise cross-correlation in an inhomogeneous moving medium," Phys. Rev. Lett. **97**, 054301 (2006).

[27]W. Heisenberg, "Die 'beobachtbaren Größen' in der theorie der elementarteilchen (Observable quantities in the theory of elementary particles)," Z. Phys. **120**, 513–538 (1943).

[28]R. Glauber and V. Schomaker, "The theory of electron diffraction," Phys. Rev. **89**, 667–671 (1953).

[29]P. L. Marston, "Generalized optical theorem for scatterers having inversion symmetry: Applications to acoustic backscattering," J. Acoust. Soc. Am. **109**, 1291–1295 (2001).

[30]H. Sato, "Green's function retrieval from the CCF of coda waves in a scattering medium," Geophys. J. Int. **179**, 1580–1583 (2009).

[31]H. Sato, "Retrieval of Green's function having coda waves from the cross-correlation function in a scattering medium illuminated by a randomly homogeneous distribution of noise sources on the basis of the first-order born approximation," Geophys. J. Int. **180**, 759–764 (2010).

[32]Y. Fan and R. Snieder, "Required source distribution for interferometry of waves and diffusive fields," Geophys. J. Int. **179**, 1232–1244 (2009).

[33]R. Snieder, K. Wapenaar, and K. Larner, "Spurious multiples in seismic interferometry of primaries," Geophysics **71**, SI111–SI124 (2006).

[34]I. Vasconcelos, R. Snieder, and H. Douma, "Reciprocity theorems and Green's function retrieval in perturbed acoustic media," Phys. Rev. E **80**, 036605 (2009).