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Are we exceeding the limits of the great circle approximation in global surface wave tomography?

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Abstract. The ray theoretical great circle approximation in global surface wave tomography is found to be limited to Earth models with a maximum degree $l \leq 30$ for surface waves at 40 s and $l \leq 20$ for surface waves at 150 s. This result holds for both phase velocity and group velocity maps. The highest resolution in present-day global surface wave tomography is close to these limits of ray theory. In order to obtain higher degree resolution models of the Earth in future surface wave tomography, it is necessary to take the scattering of surface waves into account. Increasing the data coverage in seismological networks will not improve the details of tomographic images if ray theory is still applied. It is essential to include the finite-frequency effects as well.

1. Introduction

Present-day surface wave tomography is usually based on the assumption that surface waves propagate along the ray path defined by the great circle connecting the source and receiver (e.g. Jordan 1978; Dziewonski 1984; Woodhouse and Dziewonski 1984). This is valid as long as the heterogeneities are larger than both the wavelength and the width of the Fresnel zone. With current data coverage, phase velocity maps (e.g. Trampert and Woodhouse, 1995; Ekstöm *et al.*, 1997; van Heijst and Woodhouse, 1999) and group velocity maps (e.g. Ritzwoller and Levshin, 1998) show length-scales of heterogeneity that are comparable to the width of the Fresnel zone.

In order to exceed the limits of classical ray theory in surface wave tomography, we must take into account that surface waves have a finite-period. Spetzler and Snieder (2001) apply the Rytov approximation to the acoustic wave equation to express the timeshift according to diffraction theory as a volume integration of the slowness perturbation field multiplied by a sensitivity kernel (also known as the Fréchet kernel). Spetzler *et al.* (2001) extend the wave diffraction technique of Spetzler and Snieder (2001) to a spherical geometry, thereby including scattering theory in global surface wave tomography. The surface wave scattering theory is valid for unconverted surface waves because it is based on the adiabatic assumption so that mode-conversion between different modes is not taken into account. In addition, the surface wave scattering approach is limited to a homogeneous spherical reference model which is the first step in an iterative inversion of surface wave data. In subsequent iterations, Fréchet kernels can be updated to include focussing and defocussing effects, but this is beyond current computational resources for a realistic problem. In a spherical reference model, the Fréchet kernel due to the scattering of waves propagating in two dimensions has the property that the maximum sensitivity to slowness perturbations is off-path the great circle. This result can also be found in Yomogida and Aki (1987), Yomogida (1992) and Woodward (1993) who work with the Rytov approximation, and in Marquering *et al.* (1998), Tong *et al.* (1998), Marquering *et al.* (1999), Dahlen *et al.* (2000), Hung *et al.* (2000) and Zhao *et al.* (2000) who use a linearised version of the cross-correlation function wherein single-scattering of waves is included.

It is found that the scattering of surface waves is increasingly important for increasing period and increasing source-receiver distance. Therefore, surface wave tomography for the longest periods suffers most from the use of the ray theoretical great circle approximation.

The use of scattering theory in surface wave tomography has previously been reported in the literature. For instance, Friederich *et al.* (1993, 1994) and Friederich (1998) derive a surface wave scattering theory in terms of displacement potentials, thereby extending the Born single-scattering method to multiple forward scattering and single-back scattering of surface waves. In Friederich (1998), the surface wave multiple-scattering theory is used in an inversion of Rayleigh waves between 20 s and 120 s in order to image the upper mantle structure of South Germany. Meier *et al.* (1997) apply WKBJ theory to refine the Born approximation for scattered waves that propagate through a smooth, heterogeneous reference medium. In that way, the validity of the Born approximation is enlarged, and at the same time the scattering theory is still applicable in inversion experiments. Pollitz (1998) developed a theory for the scattering of spherical elastic waves from a small spherical inclusion.

We present a synthetic experiment which shows that scattering effects are important in surface wave propagation if the Earth's structure exceeds angular degree 20 for Love waves at 150 seconds and angular degree 30 for Love waves at 40 seconds. This is close to the current limit of angular degree resolved in existing, global Earth models. If we want to increase lateral resolution in surface wave tomography in future global models, scattering effects need to be accounted for.

2. A synthetic surface wave experiment

The synthetic surface wave experiment compares the relative phaseshift $d_{ray/scat}$ due to ray theory and scattering theory, respectively. The relative phaseshift is computed

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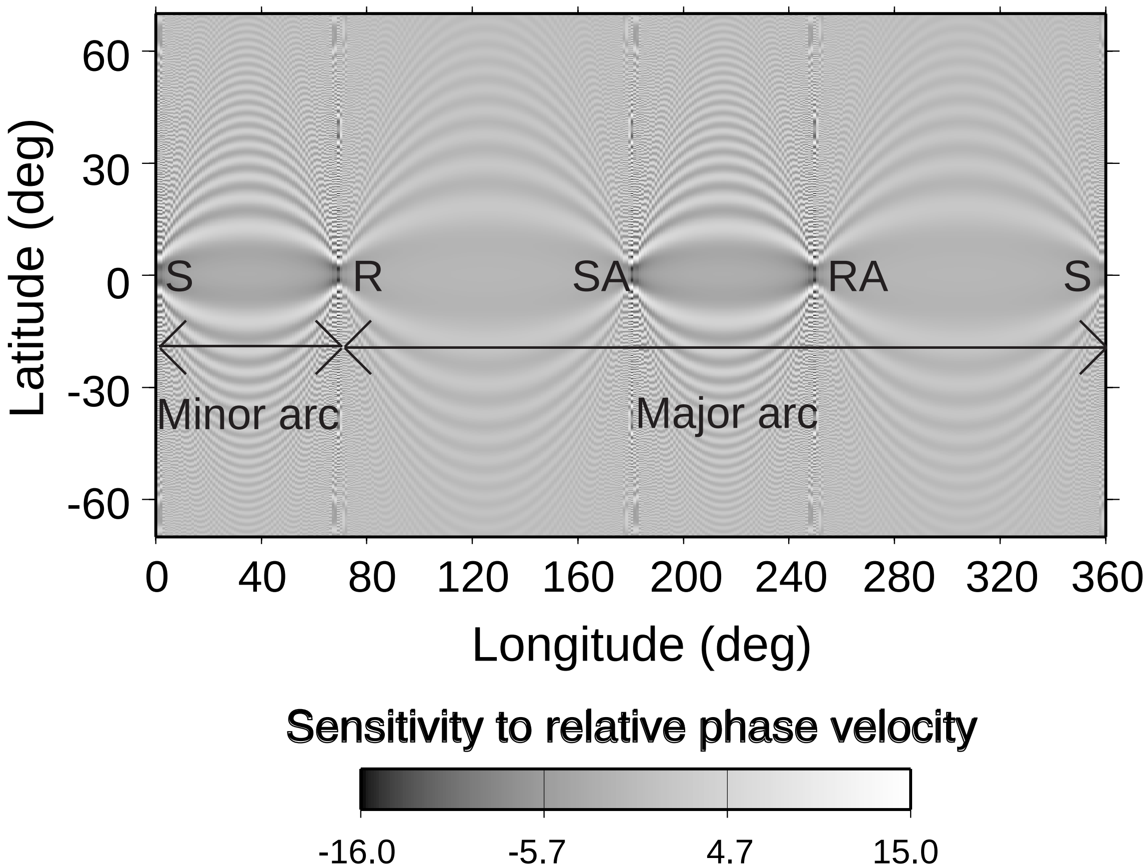

Figure 1. The scattering sensitivity kernel for relative phase velocity measurements is computed point by point on the sphere. The epicentral distance is 70° for the minor arc and 290° for the major arc. The source position is denoted by S, the receiver anti-pode position is RA, the source anti-pode position is SA and the receiver position is R. The sensitivity kernel due to scattering theory for the major arcs is constructed by three scattering sensitivity kernels for minor arcs. The sensitivity kernel for relative phase velocity perturbations is calculated for Love waves at the single period of 150 s. The sensitivity kernel has sidelobes so that the first Fresnel zone and higher order Fresnel zones are visible.

as a summation of spherical harmonic coefficients C_l^m multiplied by the kernel $K_{l,m}^{ray/scat}$ for degree l and order m derived for ray theory or scattering theory in a spherically symmetric reference model. Hence,

$$d_{ray/scat} = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{m=l} C_l^m K_{l,m}^{ray/scat}, \quad (1)$$

(see Spetzler *et al.*, 2001 for a derivation of Eq. (1)). The maximum angular degree of the expansion is denoted by l_{max}. We show in Fig. 1 the scattering sensitivity kernel for relative phase measurements of Love waves at 150 s as a function of location on the sphere. The source S is located on the equator line at latitude (0°, 0°), and the receiver R is on the equator line at (0°, 70°), which implies an epicentral distance of 70° for the minor arc and an epicentral distance equal to 290° for the major arc. The source anti-pode is denoted by SA and the receiver anti-pode by RA. The major arc scattering sensitivity kernel is constructed from three minor arc scattering sensitivity kernels; one between the source and receiver anti-pode, one between the receiver anti-pode and source anti-pode and one between the source-anti-pode and receiver, respectively. The scattering sensitivity kernel for the minor arc surface wave has the form of the Fresnel zone for a point source which has the maximum width at half epicentral distance. Note that the sidelobes do not vanish if we take the frequency averaging, inherent to the measurement (5 mHz), into account. In contrast, the ray-geometrical sensitivity kernel is only nonzero on the great circle that joints the source and receiver which in the case of Fig. 1 is the equator line. When the background medium is assumed to be homogeneous, the ray reference linking the source and receiver is a straight line on the sphere. In a heterogeneous reference medium, ray bending effects will further complicate matters (e.g. Pollitz, 1994, Meier *et al.*, 1997).

The kernels $K_{l,m}^{ray/scat}$, such as in Fig. 1., are projected on spherical harmonics of angular degree l and order m, yielding the ray theoretical or scattering theoretical kernels in Eq. (1).

The surface wave scattering theory in Eq. (1) is based on the first order Rytov approximation which has a much larger validity than the ray theoretical approach. In Fig. 4 of Spetzler and Snieder (2001), a finite-difference experiment shows that the scattering theory derived from the first order Rytov approximation predicts the 'observed' arrival times of waves well even for length-scales of heterogeneity that are smaller than the width of the Fresnel zone. In contrast, ray theory breaks down when the characteristic length of inhomogeneity is comparable with the width of the Fresnel zone.

The limit of the ray-theoretical great circle approximation in surface wave tomography is estimated by using a synthetic experiment. The source and receiver positions are from the recent dataset of relative phaseshifts of Trampert and Woodhouse (2001). Surface wave phaseshifts using ray theory and scattering theory are calculated for Love waves at 40 and 150 seconds. The discrepancy between ray theory and scattering theory is defined as the averaged relative error between the respective phaseshifts. Hence,

$$\text{rel. error} = \frac{100\ \%}{N} \sum_{i=1}^{N} \left| \frac{d_i^{ray} - d_i^{scat}}{d_i^{scat}} \right|, \qquad (2)$$

where N is the number of source-receiver geometries. To avoid numerical instability, source-receiver pairs with $|d_i^{scat}| \leq 1 \times 10^{-3}$ have not been included in Eq. (2). The input model consist of randomly drawn spherical harmonic coefficients for each degree. The velocity perturbation is set to 10 %. The degree l of the input model varies from 1 to 40 corresponding to the size L of velocity heterogeneity between 40000 km and 1000 km, respectively. The ray theoretical approach based on the great circle approximation and the first order scattering theory are both linear theories, so the amplitude of the velocity perturbation does not influence the relative error in Eq. (2). Thus, for realistic Earth models with either a white or a red spectrum, the synthetic experiment presented in this paper indicates to which extent the ray theoretical great circle approximation differs from a more exact scattering theory.

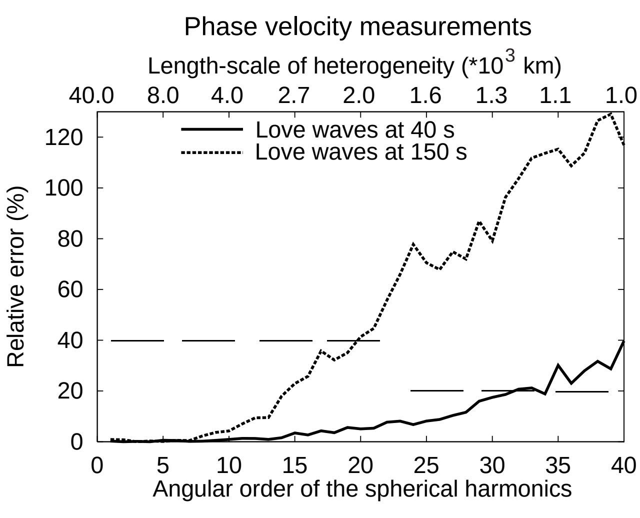

Figure 2. The synthetic experiment for phase velocity measurements showing that the relative error between data computed with scattering theory and ray theory increases for decreasing characteristic length of velocity anomalies in a global surface wave experiment for Love waves between 40 s and 150 s. The length-scale of heterogeneity is expressed in the angular degree l ranging between 1 ($L \approx 40000$ km) and 40 ($L \approx 1000$ km). The source-receiver positions in the Love wave dataset from Trampert and Woodhouse (2001) are applied. The horizontal long-dashed lines indicate the observed relative error for the Love wave dataset for Love waves at 40 s and 150 s.

3. Results

The relative error due to the ray theoretical great circle approximation is presented in Fig. 2 in case of Love waves at 40 s and 150 s. The horizontal axis shows the length-scale of velocity perturbations expressed by the spherical harmonic angular degree varying from 1 (length-scales of velocity anomalies $L \approx 40000$ km) to 40 ($L \approx 1000$ km). The vertical axis of Fig. 2 is the averaged relative error in percent for Love waves at 40 s (solid line) and at 150 s (dashed line).

Given the same angular degree of velocity perturbation, the discrepancy between phase velocity maps using ray theory and scattering theory is smaller for Love waves at 40 s than at 150 s. This is because the forward scattering of surface waves is most important for the longest periods.

Suppose one wants to invert phase velocity measurements in a global surface wave experiment using the ray theoretical great circle approximation. The scattering theory being more exactly, the error due to the great circle approximation should not exceed the relative error in the phase measurements themselves. This measured relative error corresponds to approximately 20 % for Love waves at 40 s and 40 % for Love waves at 150 s (Trampert and Woodhouse, 2001). Fig. 2 tells us then for which angular degree l the ray-geometrical inversion is acceptable. A ray-theoretical tomographic surface wave inversion based on the great circle approximation for Love waves at 40 s and 150 s is limited to angular degrees smaller than $l = 30$ and $l = 20$, respectively. These limits are close to the highest resolution in present-day global surface wave tomography using phase velocity measurements. The same result holds for group velocity maps for Love waves at period of 40 s and 150 s.

4. Discussion and Conclusions

The application of the ray-theoretical great circle approximation in global surface wave tomography is limited to structures of the Earth with the length-scale of velocity perturbations which is close to the highest resolution of Earth structure obtained in present-day global surface wave tomographic inversions. The error introduced by the use of the ray theoretical great circle approximation in global surface wave tomography is significant for the length-scale of heterogeneity expressed in angular degree $l \geq 30$ (or $L \leq$ 1300 km) and $l \geq 20$ (or $L \leq 2000$ km) for Love waves at 40 s and 150 s, respectively. For this reason, it is important to incorperate the scattering of surface waves in future global surface wave tomography. It is the only way to obtain global models of the Earth with a higher resolution than is currently possible.

The developed scattering theory for surface waves is a linear theory which relates surface wave phase velocity or group velocity measurements to the velocity perturbation structure of the Earth. The surface wave scattering theory is applicable to unconverted minor arc and major arc surface waves in an inversion based on a constant spherically symmetric background model. It is just as easy to use first order scattering theory than the ray-theoretical great circle approach in the inversion of surface wave measurements. In a second stage, it can be envisaged to update the kernels in a 3D background model. This would take bending, focussing and defocussing of the wavefronts into account.

Theoretically this is not more difficult, but still limited by computational reasons.

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