What is a wave?

Most people assume that a wave, being central to all the phenomena we observe, has a uniform definition. But defining this basic concept isn't so easy.

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Ask your colleagues and students to define a wave, and you may be surprised at the answers you get. Even wave professionals are prone to confusion and vagueness when confronted with such an apparently simple question. Students often begin circularly: "a wave is a solution to the wave equation." But what is a wave equation? Professionals are more likely to mutter something about propagation speed, as if smell and heat didn't propagate with some speed. Mathematicians tend to give formal characterizations based on the hyperbolicity of certain differential equations.

Just as a definition of noise must be grounded empirically, so to define a wave we should look at what nature has to offer. A preliminary answer might be: a wave is a propagating imbalance. The imbalance concept is also present in a simple oscillator where kinetic energy and potential energy are interchanged during the oscillation. Hamilton's principle (that the path taken by a dynamical system is the one that minimizes the time integral of the difference between the kinetic and potential energies) is a formalization of this idea of interchange between the two forms of energy. However, what makes a wave different from a single oscillator is that this imbalance propagates. (Of course, two travelling waves can form a standing wave, but let us ignore this complication for the moment.)

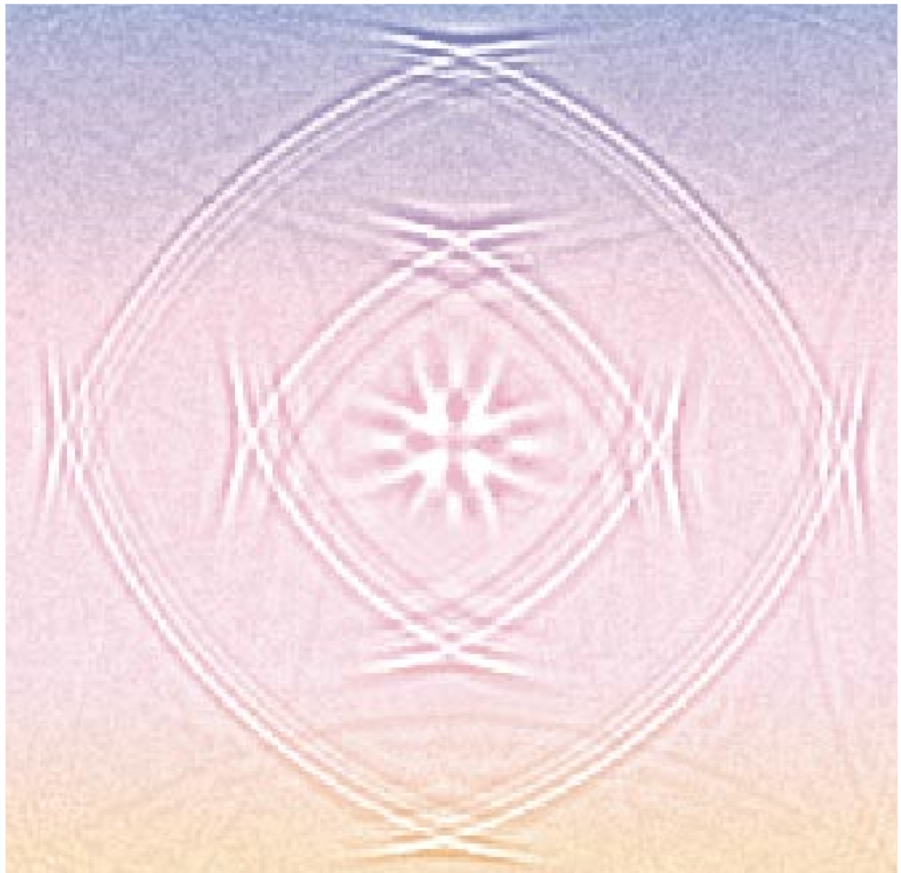

Figure 1 **The recorded wavefield that has propagated through a homogeneous silicon crystal at three times after excitation at the other side of the crystal. Details of the experiment are given by Wolfe[3]. This example shows that wavefronts in a homogeneous medium can be square rather than round. (Courtesy of J. P. Wolfe.)**

Stable equilibria

At this simplest of levels, the ubiquity of (classical) waves can be attributed to nature's love of stable equilibria. Whatever the forces that connect bits of matter together (electromagnetic or gravitational, for instance), for small perturbations about a stable equilibrium point, the forces are approximately linear; a linear restoring force implies harmonic oscillation; and coupled systems of oscillators support both propagating and standing disturbances. Linearity also implies superposition, so we can carefully add periodic solutions together to get finite wave 'packets'. So, for small perturbations about an equilibrium state of coupled or extended systems, waves are the natural consequence of the stability of simple harmonic motion.

The miracle of the waves that we see is the organization they display, but there are examples where this organization is destroyed. Strong scattering leads to diffusive behaviour rather than wave propagation. The scatterers destroy the level of organization in the incident wave and ultimately lead to diffusive (un-wave-like) behaviour. Similarly, when a wave breaks on a beach, the advective terms in the equation of motion couple all the different length scales in the wave, and the organization we see in the swell is destroyed. Ultimately, the wave is dissipated as heat. In view of these examples, where a wave ceases to be a wave because of the destruction of its degree of organization, we are led to modify our definition of a wave to become: a wave is an organized propagating imbalance.

Wave propagation is in many situations described by a linear differential equation. In reality, nonlinearity is important, and this nonlinearity may destroy the waves. As an example, consider the waves on the beach again. Look far out at the ocean from any beach, and you will see ripples on the surface of the water with a period of 5–10 seconds. As these ripples approach the beach, their heights increase until they can no longer support their own weight and they break catastrophically. Mathematically, this is caused by the nonlinear terms in the equation of motion becoming increasingly important as the waves grow. When nonlinearity becomes important, organized wave motion changes into turbulent motion. In this process, it is impossible to state exactly at which point the wave ceases to be a wave.

Nonlinearity is sometimes essential for maintaining the organization of a wave. In solitons, the wave spreading by dispersion is exactly (and miraculously) offset by the nonlinear steepening of the wave, so that a solitary wave maintains its identity. This means that nonlinearity can lead to the creation of organization as well as to its destruction.

The simplest soliton to produce is the

cylindrical hydraulic jump. Go into the kitchen and turn on the tap at the kitchen sink. As the water strikes the sink, its vertical momentum is converted into horizontal momentum. In most cases, when the water first hits the surface, it is travelling faster than the speed of surface-water waves, so disturbances cannot propagate as surface waves and are swept downstream by the water. But the water must slow down, and at some point it slows to the speed of surface-water waves. What happens then is truly remarkable. A jump, or shock, develops — the thickness of the water increases almost discontinuously. Further downstream, the water's surface is awash with surface waves that are now free to propagate. But the jump is stationary, so why should we regard this as a wave-like phenomenon? Well, imagine you are in a boat being swept downstream by the water. In your frame of reference the hydraulic jump is a solitary 'wave' racing upstream, much like a tidal bore.

Even the ripples of space–time are waves.

Diffusion

To return to the original question, many people may say something about wave-like versus diffusive behaviour. In many physical phenomena, there is no clear distinction between these two extremes of behaviour. For instance, take light propagating in a turbid medium such as milk. The turbidity is the result of scattering. (The absorption cross-section of the fat molecules in the milk is much smaller than the scattering cross-section — the opposite of ink, for instance.) The equations governing the electric field are still the same linear-wave equations that follow from Maxwell's equations. But what the eye registers is not the electric field itself, but rather the intensity of the field. Because the field reaching the eye is the superposition of the uncountably many scattered waves originating in the milk (the equations are linear), the actual intensity is the intensity of this superposition. It is easy to see that this total intensity has both a coherent and an incoherent term in the superposition.

If the different scattered waves do not interfere constructively with one another, then the total intensity is merely the sum of the intensities of the individual waves. If these individual, non-interfering scattering terms are thought of as representing a vast number of uncorrelated brownian paths through the milk, it is no surprise that the no-interference intensity satisfies a diffusion equation (which is the equation of the probability distribution for brownian motion). So the electric field satisfies a wave equation, as Maxwell said it must, but the quantity we measure (the intensity) satisfies a diffusion equation.

Some phenomena are clearly diffusive, with no wave-like implications — heat, for instance. We all 'know' that heat conduction is governed by the diffusion equation. Maxwell actually had his doubts about this (see ref. 1 for a fascinating account). The standard diffusion equation doesn't take into account any propagation speed, so it cannot really be a fundamental description of the transport of heat; according to this equation, if you apply a heat source to one end of a rod, the temperature at the other end begins to change instantaneously! Maxwell, working from kinetic theory, imported a ballistic term into the equations of heat conduction. He ended up with the telegraph equation (it has first and second derivatives with respect to time), with its trade-off between the diffusive behaviour (which comes from the first time derivative) and ballistic behaviour (coming from the second time derivative). Maxwell dropped this ballistic term after concluding[2] that it "may be neglected, as the rate of conduction will rapidly establish itself".

That was consistent with experiments 100 years ago, but not any longer. As far back as the 1960s, ballistic heat pulses were observed at low temperatures. The idea is that heat is just the manifestation of microscopic motion. Computing the classical resonant frequencies of atoms or molecules in a lattice gives numbers of the order of 10^{13} Hz, that is, in the infrared, so when molecules jiggle they give off heat. These lattice vibrations are called phonons. Phonons have both wave-like and particle-like aspects. Lattice vibrations are responsible for the transport of heat, and we know that heat is a diffusive phenomenon. However, if the lattice is cooled to near absolute zero, the mean-free scattering path of the phonons becomes comparable to the macroscopic size of the sample. When this happens, lattice vibrations no longer behave diffusively but are actually wave-like. By controlling the temperature of a sample, one can control the extent to which heat is ballistic (wave-like) or diffusive. In essence, if a heat pulse is launched into such a sample (by passing a current through a wire, for instance), and if the phonons can get across the sample without scattering, they will propagate like waves. The more they scatter, the more diffusively they behave. When it's very cold, heat waves propagate as waves. Figure 1 shows an example; many more are found in ref. 3.

Waves are not only elusive in their character, their presence is ubiquitous in nature. Our two main senses, vision and hearing, rely on waves. We call these the 'main' senses because they give us the most precise information about the environment. It is typical that there are common-language words for the loss of eyesight or hearing, but not for the loss of sense of smell, taste and warmth. Most of what we know about the world around us we learn through waves. In addition, neurons work by the propagation of electric waves through the axons. A prime example is the triggering of the heart by a propagating electric pulse through the heart tissue.

Even the ripples of space–time are waves. These are called gravitational waves and propagate at the speed of light. The first observatories with a real chance of detecting gravitational waves are now coming online. These instruments, enormously long interferometers, will be able to measure strains on the order of 10^{-20} or smaller.

Quantum mechanics

Another field where waves have a central role is quantum mechanics, from which we learn that everything has a wave character. Einstein used the relation $E = hf$ (energy equals Planck's constant times frequency) to connect the wave frequency of light with the energy of light's discrete quanta (photons). De Broglie extended this to electrons and other ponderable matter. For classical waves, dissipation generally damps the wave motion, and ultimately everything seems to come to rest. Quantum mechanics shows that matter waves do not suffer from dissipation. Even the ground state of the harmonic oscillator is in harmonic motion. Matter waves never come to rest. Taking this last idea a step further, one can conjecture that the ubiquity of waves is crucial to our concept of time. Change is the manifestation of time, and regular oscillations are a clear manifestation of change. Appropriately, the waves that propagate in quartz crystals are now the dominant tool used to keep track of time.

It is clear that waves are ubiquitous in nature and that they are central to the structure of matter and time as well as to many physical, biological and chemical phenomena. It is striking that the concept of waves is so hard to define, and that the distinction between wave-like and non-wave-like behaviour can be so fuzzy. Taking all these examples into account, we stick with our definition of a wave as an organized propagating imbalance; just don't ask us to define 'organized'. ■

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