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An alternative parameterisation for surface waves in a transverse isotropic medium

Everhard Muyzert *, Roel Snieder

Department of Geophysics, Utrecht University, P.O. Box 80021, 3508 TA Utrecht, Netherlands

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Abstract

The elastic properties of the Earth can only be retrieved by seismic waves when assumptions about the seismic anisotropy are made. One such an assumption is to assume that the Earth is transverse isotropic. In a transverse isotropic Earth Love waves are sensitive to two independent elastic parameters and Rayleigh waves to four parameters. In addition these waves are sensitive to density. However, resolving four elastic parameters together with density as function of depth from Rayleigh waves phase velocity curves is difficult as the large number of parameters can make the inverse problem easily under-determined. We show that the partial derivatives of fundamental and higher mode Rayleigh wave phase velocities with respect to the parameters of transverse isotropy for periods $20 < T < 200$ s are very similar to each other. This means that the inversion of Rayleigh phase velocity data has intrinsic trade-offs. We show that Rayleigh wave data can only resolve three combinations of parameters of the transverse isotropic medium: the S-velocity β_V, the sum of the horizontal and vertical P-velocity $\alpha_+ = 1/2[\alpha_H + \alpha_V]$ and their difference $\alpha_- = 1/2[\alpha_H - \alpha_V]$. The influence of α_- is limited to only the upper 100 km Earth. To take full advantage of this parameter set the data-set should consist of both fundamental and higher mode data with periods between $20 < T < 200$ s. For this parameter set the influence of the density is small and we think it cannot be resolved when realistic variations in all parameters are considered. For Love waves the only relevant parameter is β_H as β_V and ρ cannot be resolved. We support our conclusions by a synthetic experiment where the bias in the inversion of Rayleigh wave phase velocity is investigated when the data is inverted for the reduced alternative parameter set.

Keywords: Earth; Surface waves; Transverse isotropic medium; Elastic; Seismic waves; Anisotropy; Transverse isotropy

1. Introduction

Seismic wave propagation in the Earth is determined by the elastic properties and density of the Earth. Most seismological studies only consider variations in the isotropic P- and S-velocities. However, since Hess (1964) provided evidence for anisotropy in the oceanic lithosphere, seismologists have determined the anisotropic properties of the Earth, an overview of such studies is given by Babuska and Cara (1991). Surface waves are a primary tool for the investigation of the Earths anisotropic properties. Transverse isotropy was, for example, required to

* Corresponding author. Schlumberger Cambridge Research, High Cross, Madingley Road, Cambridge CB3 0EL, UK. Tel.: +44-1223-325432; fax: +44-1223-361473; e-mail: muyzert@cambridge.scr.slb.com

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explain the Love–Rayleigh discrepancy present in the surface wave data used for the PREM model (Dziewonksi and Anderson, 1981). Transverse isotropy is a simple type of anisotropy which has only one symmetry axis in either the vertical or horizontal direction. Transverse isotropy has also been observed in the oceanic lithosphere (Cara and Leveque, 1987, 1988; Nishimura and Forsyth, 1989). Surface waves have also been used to map the large scale variations in the azimuthal properties of the Earth (Tanimoto and Anderson, 1985; Montagner and Tanimoto, 1991). Inversions of surface wave data for the density structure have been carried out by Nolet (1977), Cara et al. (1984) and Tanimoto (1991).

Determination of the full elasticity tensor c_{ijkl} with its 21 independent parameters in the Earth is beyond the resolving power of the seismic data. Analysis of the partial derivatives of phase velocity has shown that there are only 13 combinations of elastic parameters involved (see Smith and Dahlen, 1973; Crampin, 1984; Montagner and Nataf, 1986). However, for realistic petrological models of the upper mantle strong correlations between the 13 combinations of elastic parameters do exist (Montagner and Anderson, 1989). For data along a given source–receiver path the number of independent parameters is even smaller (Maupin, 1985). In that case the partial derivatives for the general anisotropic medium can be written as a linear combination of those for the transverse isotropic medium with vertical symmetry axis. This means that for a given source–receiver path, Rayleigh waves are sensitive to only four combinations of elastic parameters and density while Love waves sample only two independent combinations of elastic parameters and density.

In this paper, we show that a well-posed inversion of fundamental and higher mode Love and Rayleigh phase velocity data with periods between $20 < T < 200$ s for transverse isotropy requires a further reduction of the number of independent parameters. For fundamental mode data this observation is not surprising as one has typically measurements at 10–15 periods that are a function of the depth variation of four elastic parameters and density. The parameters in such an inversion are better constrained when higher modes are added to the data-set. However, the parameters for transverse isotropy have inherent trade-offs. Leveque and Cara (1985) showed that for such data a consistent inversion can be carried by inducing non-diagonal terms in the a priori covariance matrix. The structure of this a priori covariance matrix is however largely subjective (Leveque and Cara, 1985).

We propose an alternative parameter set that does not have these trade-offs. This alternative parameter set is built from linear combinations of the elastic parameters in such a way that it minimises the trade-offs. We investigate the relative importance of these parameters in combination with density and show that we can resolve only three combinations of parameters instead of $4 + 1$ parameters. We illustrate this statement with an experiment in which we invert phase velocity data, computed for the full parameter set, for the reduced alternative parameter set.

2. Transverse isotropy

Wave propagation in transverse isotropy is sensitive to 5 independent elastic parameters (A, C, L, N, F) and density (see Love, 1927). In Table 1 the definition of this so-called Love parameter set is given in terms of seismic velocities, the fourth order elasticity tensor c_{ijkl} and its shorthand notation C_{ij}. The interpretation of the elastic parameters A, C, L, N is straightforward, that of F is more complicated. By definition, $F = c_{3311} = c_{3322}$ which indicates that F is related to the velocity of a wave propagating in the vertical plane between source and receiver. Throughout this paper we denote this velocity by γ. Anderson (1961) has given an expression for F as function of a wave at 45° incidence of the vertical axis and

Table 1
Definition of the Love parameters

Love (1927) parameters	Velocity	Elasticity tensor	Matrix notation
A	$\rho\alpha_H^2$	c_{1111}	C_{11}
C	$\rho\alpha_V^2$	c_{3333}	C_{33}
L	$\rho\beta_V^2$	$c_{2323} = c_{1313}$	$C_{44} = C_{55}$
N	$\rho\beta_H^2$	c_{1212}	C_{66}
F	$\rho\gamma^2$	$c_{3311} = c_{3322}$	$C_{31} = C_{32}$

introduced the parameter $\eta \equiv F/(A-2L)$ (here η is given in the notation of Takeuchi and Saito (1972) which is the inverse of Anderson's definition). In isotropy $F = A - 2L$ and $\gamma^2 = \alpha^2 - 2\beta^2$.

Linearised inversion of surface wave dispersion measurements can be carried out using the partial derivatives of the phase velocity with respect to the model parameters. For Love waves such an inversion

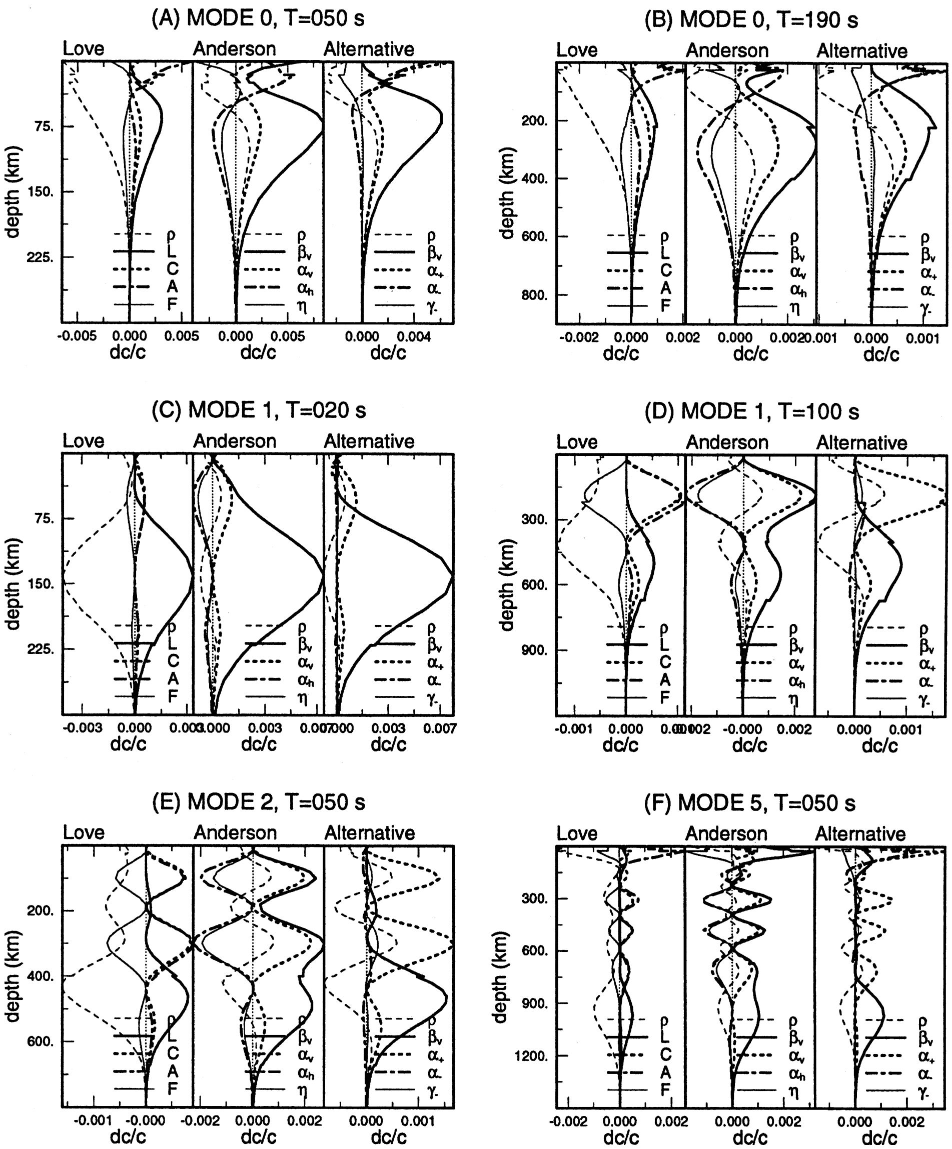

Fig. 1. Partial derivatives of the Rayleigh wave phase velocity for three parameter sets. The left panels show those for the Love parameter set, the middle panel shows those for the Anderson parameter set and alternative parameter set is shown in the right panel. Linestyles for the partial derivatives with respect to the various parameters are indicated in the figure. (A) Fundamental mode at $T = 50$ s. (B) Fundamental mode at $T = 190$ s. (C) First higher mode at $T = 20$ s. (D) First higher mode at $T = 100$ s. (E) Second higher mode at $T = 50$ s. (F) Fifth higher mode at $T = 50$ s.

is based on Eq. (1) which relates the relative variation in phase velocity $[\delta c/c]$ to integrals of the variations in the medium with depth.

$$\left[\frac{\delta c}{c}\right] = \int_{z=0}^{R} \frac{L}{c}\left[\frac{\partial c}{\partial L}\right]\frac{\delta L}{L}\,\mathrm{d}z + \int_{z=0}^{R} \frac{N}{c}\left[\frac{\partial c}{\partial N}\right]\frac{\delta N}{N}\,\mathrm{d}z + \int_{z=0}^{R} \frac{\rho}{c}\left[\frac{\partial c}{\partial \rho}\right]\frac{\delta \rho}{\rho}\,\mathrm{d}z \qquad (1)$$

In Eq. (1) the terms, $[\partial c/\delta L]$, $[\partial c/\delta N]$, $[\partial c/\delta \rho]$ are the partial derivatives to the model parameters. The unit of the terms like L/c $[\partial c/\partial L]$ are reciprocal length. Details about the computation of the partial derivatives can be found in Takeuchi and Saito (1972). For Rayleigh waves a relation similar to Eq. (1) but with terms for (A,C,L,F,ρ) applies.

The partial derivatives of the Rayleigh waves for the Love parameter set at six frequencies and modes are shown in the left panels of Fig. 1. These partial derivatives have been computed for the equivalent isotropic PREM model (Dziewonksi and Anderson, 1981). An important observation made from Fig. 1 is that the higher mode partial derivatives for $[\partial c/\partial A]$, $[\partial c/\partial C]$ and $[\partial c/\partial F]$ all have an almost identical shape but different amplitude. Because of this similar shape it is not possible to resolve the parameters A, C, and F independently using higher mode Rayleigh wave data alone. This is in contrast to the fundamental mode partial derivatives for A, C, and F that have significant differences in the upper 100 km. Another interesting observation is that for higher modes, $[\partial c/\partial A]$, $[\partial c/\partial C]$, $[\partial c/\partial F]$ are more sensitive to structure at shallower depths than $[\partial c/\partial L]$. This is due to the fact that the partial derivative for L is determined by S-motion while those for A, C and F are determined by P-motion. The partial derivative for the density is a mixture of the P- and S-motion and has a negative sign.

Anderson (1961) presented a different parameter set for transverse isotropy (α_H, α_V, β_H, β_V, $\eta \equiv F/(A-2L)$). It follows from differentiation of the Love parameter to velocity (see Table 2). This parameter set has been used for the construction of the PREM model (Dziewonksi and Anderson, 1981). The partial derivatives for the Anderson parameter set are shown in the middle panels of Fig. 1.

The partial derivatives $[\partial c/\partial \alpha_V]$ and $[\partial c/\partial \eta]$ are very similar to those for C and F, respectively. The partial derivatives for $[\partial c/\partial \beta_V]$ and $[\partial c/\partial \alpha_H]$ are very different from $[\partial c/\partial L]$ and $[\partial c/\partial A]$ which is the result of Anderson's definition of $F \equiv \eta(A-2L)$ (see Table 2). Fig. 1 shows that for the fundamental mode $[\partial c/\partial \alpha_H]$ has a zero crossing. For the higher modes it becomes negative and $[\partial c/\partial \alpha_H] \approx -[\partial c/\partial \alpha_V]$. Thus for the higher modes an increase in horizontal P-velocity gives a decrease in phase velocity. Because of the great similarity in the shape $[\partial c/\partial \alpha_H]$ and $[\partial c/\partial \alpha_V]$ it will be very difficult to resolve α_H and α_V independently in an inversion. In the Anderson parameterisation the partial derivative for density $[\partial c/\partial \rho]$ has a different shape and is oscillating around zero. This is the result of the differentiation of the elastic parameters that are a function of density.

Table 2
Definition of the partial derivatives of Rayleigh waves in the Love, Anderson and alternative parameterisation

Love (1927)	Anderson (1961)	Alternative
$[\partial c/\partial A]$	$[\partial c/\partial \alpha_H] = [\partial c/\partial A][\partial A/\partial \alpha_H] + [\partial c/\partial F][\partial F/\partial \alpha_H]$	$[\partial c/\partial \alpha_+] = 1/2[\partial c/\partial \alpha_H] + 1/2[\partial A/\partial \alpha_V]$
$[\partial c/\partial C]$	$[\partial c/\partial \alpha_V] = [\partial c/\partial C][\partial C/\partial \alpha_H]$	$[\partial c/\partial \alpha_-] = 1/2[\partial c/\partial \alpha_H] - 1/2[\partial A/\partial \alpha_V]$
$[\partial c/\partial L]$	$[\partial c/\partial \beta_V] = [\partial c/\partial L][\partial L/\partial \beta_V] + [\partial c/\partial F][\partial F/\partial \beta_V]$	$[\partial c/\partial \beta_V] = [\partial c/\partial L][\partial L/\partial \beta_V]$
$[\partial c/\partial N]$	$[\partial c/\partial \beta_H] = [\partial c/\partial N][\partial C/\partial \beta_H]$	$[\partial c/\partial \beta_H] = [\partial c/\partial N][\partial C/\partial \beta_H]$
$[\partial c/\partial F]$	$[\partial c/\partial \eta] = [\partial c/\partial F][\partial F/\partial \eta]$	$[\partial c/\partial \gamma_-] = [\partial c/\partial F][\partial F/\partial \gamma] - 0.7[\partial c/\partial \alpha_+]$
$[\partial c/\partial \rho]_L$	$[\partial c/\partial \rho]_A = [\partial c/\partial \rho]_L + [\partial c/\partial A][\partial A/\partial \rho] + [\partial c/\partial C][\partial C/\partial \rho] + [\partial c/\partial L][\partial L/\partial \rho] + [\partial c/\partial F][\partial F/\partial \rho]$	$[\partial c/\partial \rho]_A = [\partial c/\partial \rho]_L + [\partial c/\partial A][\partial A/\partial \rho] + [\partial c/\partial C][\partial C/\partial \rho] + [\partial c/\partial L][\partial L/\partial \rho] + [\partial c/\partial F][\partial F/\partial \rho]$

For Love waves in a transverse isotropic medium only the partial derivatives to the elastic parameters, L and N and density are non-zero. In the left panel of Fig. 2 we show the partial derivatives for these parameters at the same frequencies and modes as the Rayleigh partial derivatives displayed in Fig. 1. The partial derivative for the density has a similar shape and amplitude but opposite sign as $[\partial c/\partial N]$. These partial derivatives have a much larger amplitude than $[\partial c/\partial L]$ of which the influence on the Love wave phase velocity is very small. In Anderson's parameter set the density partial derivative is negligible for the fundamental mode and only small for the longer periods higher modes (e.g., the fifth higher mode at 50 s) (see right hand panel of Fig. 2).

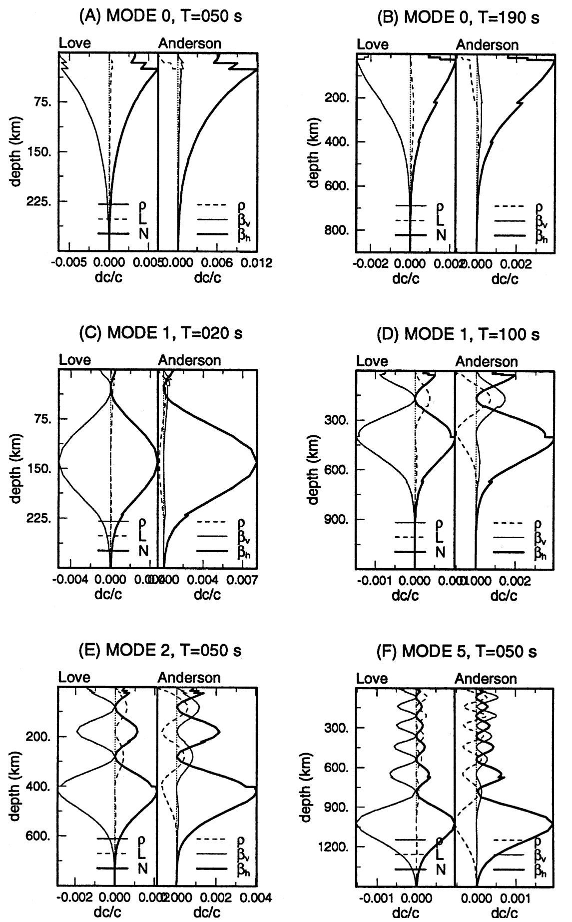

Fig. 2. Partial derivatives of the Love wave phase velocity for the Love parameter set (left panel) and the Anderson parameters (right panel).

3. Alternative parameterisation

The analysis of the partial derivatives for the transverse isotropic medium has shown that it is unlikely that all parameters can be resolved from phase velocity data. The Love parameter set has some inherent trade-offs present in the parameters A, C, and F. The Anderson parameter set has trade-offs in α_V, α_H, β_V and η due to the definition of $F = \eta(A - 2L)$. On the other hand, for the Anderson parameter set the influence of the density is much smaller. We therefore propose an alternative parameter set which is more suitable for the inversion of surface wave data (see Table 2). We use the alternative definition of $F \equiv \rho\gamma^2$ by which no additional terms enter the partial derivatives for α_H and β_V that causes undesirable trade-offs. Using this definition the partial derivatives for α_H, α_V, β_H, β_V and γ have a shape similar to those for the Love parameters A, C, L, N and F. The partial derivative for density has the same shape as Anderson's density partial derivative. As $[\partial c/\partial \alpha_H]$, $[\partial c/\partial \alpha_V]$ and $[\partial c/\partial \gamma]$ have a similar shape we define three new parameters: the average of the horizontal and vertical P-velocity $\alpha_+ = 1/2[\alpha_H + \alpha_V]$, the difference between the P-velocities $\alpha_- = 1/2[\alpha_H - \alpha_V]$ and the difference $\gamma_- = [\gamma - 0.7\alpha_+]$. The factor 0.7 in the definition of γ_- accounts for amplitude differences between $[\partial c/\partial \gamma]$ and $[\partial c/\partial \alpha_+]$ and is empirically determined as no analytical relation could be derived.

The partial derivatives for the alternative parameter set (β_V, α_+, α_-, γ_-, ρ) are shown in the right panels of Fig. 1. Those of the parameters β_V, α_+, ρ have the largest amplitudes and have very different shapes. We expect therefore that it is possible to resolve these parameters independently. Note that the partial derivative for ρ is identical to Anderson's. The higher modes partial derivatives for α_- and γ_- have a small magnitude and it is therefore unlikely that they can be resolved. For the fundamental mode the partial derivative for γ_- has a larger amplitude but still it is small and is likely to have a trade-off with α_- which depth penetration is also limited to the upper 200 km.

We illustrate the importance of the elastic parameters on the phase velocity by the following example. We perturb the parameters (β_V, α_+, α_-, γ_-, ρ) in the upper 220 km by 1%, below this depth the model is not perturbed. The corresponding phase velocity perturbations with respect to the PREM model have computed using Eq. (1). For the Rayleigh waves the results are shown in Fig. 3. The influence of β_V on the phase velocity is large as a 1% increase can give a 0.8% change in $[\mathrm{d}c/c]_{\beta_V}$. As the modes penetrate deeper in the Earth with increasing period and mode, the effect of a 220-km thick perturbed β_V layer on the fifth overtone is only observable at periods smaller than 20 s. The partial derivative for α_+ penetrate less deep than β_V at the same period thus the variations in $[\mathrm{d}c/c]_{\alpha_+}$ occur at larger periods than for β_V. The influence of a 1% perturbation in α_- and γ_- on the phase velocity is small. We think that it is impossible to retrieve the α_- and γ_- independently because they are both predominantly sensitive to the crust and lithosphere. The sensitivity of γ_- is less then α_- and therefore we propose to neglect this parameter in the inversion. We cannot also neglect α_- as in the crust and lithosphere large variations ($>10\%$) in the elastic properties occur.

The influence of the density perturbation on the phase velocity is very small (see Fig. 3). This does not contradict with Tanimoto (1991) who retrieved density information from surface waves. Tanimoto used long period data with $T > 100$ s and inverted the data-set only for S-velocity and density. Tanimoto found global density variations in mantle of about 0.5% while β_V varied 7%. Fig. 3 shows that at these long periods the perturbed S-velocity and density give a similar phase velocity perturbation.

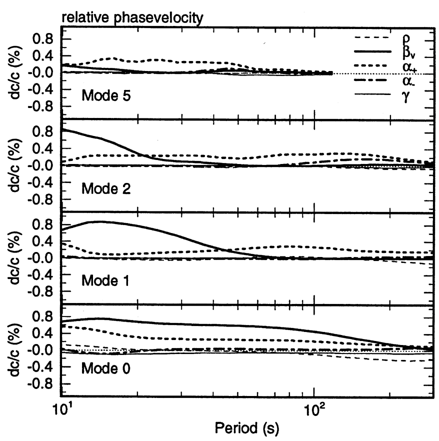

Fig. 3. Relative Rayleigh wave phase velocity perturbations $\mathrm{d}c/c$ as function of period for increase in each model parameter of 1% in the upper 220 km.

There are two reasons why the density can be neglected in the inversion of phase velocity data for periods between $20 < T < 200$ s. First, the oscillatory nature of $[\partial c/\partial\rho]$ makes it very difficult to resolve the smooth structure of the density profile. In addition, seismologists normally put smoothness constraints on the inversion of phase velocity data that make it impossible to find rough density profiles. Second, the changes in the density of the Earth are likely to be smaller than the variations in the elastic parameters. In the continental lithosphere the density can vary up to 3% due to variations in mantle temperature (Vlaar et al., 1994). The corresponding variation in phase velocity is much less than the observed variations in the phase velocity in the lithosphere. In the sub-lithospheric mantle the density variations are likely to be smaller. Subducting slabs are probably the most important source for density variations in the mantle. We estimate the maximum density contrast in a slab in the following way. The centre of a slab at 400 km depth has a temperature contrast of 800° with the surrounding mantle (Helffrich et al., 1989; De Jonge, 1995). Given a thermal expansion coefficient for the mantle of $\alpha = 3.510^{-5}$ K^{-1} a maximum density contrast of 3% can be expected. This means that the expected density contrasts in the Earth are smaller than the variation in the elastic parameters. A constant density variation of 3% gives a 0.75% variation in the phase velocity (see Fig. 3). As phase velocity measurement errors are of the order of 1% this is still within the uncertainties (Nakanishi and Anderson, 1984; Van Heijst et al., 1994). In conclusion, we think that it is unlikely that one can resolve the density for periods between $20 < T < 200$ s given the lack of sensitivity to smooth profiles, measurement errors and variations in other parameters.

For the sake of completeness we show in Fig. 4 the sensitivity of the Love wave phase velocity. The sensitivity of the Love waves to perturbations in β_H is large, a 1% perturbation in β_H results in a $[\mathrm{d}c/c]_{\beta_H}$ of 1%. The sensitivity to density and β_V perturbations is much smaller, by which we do think

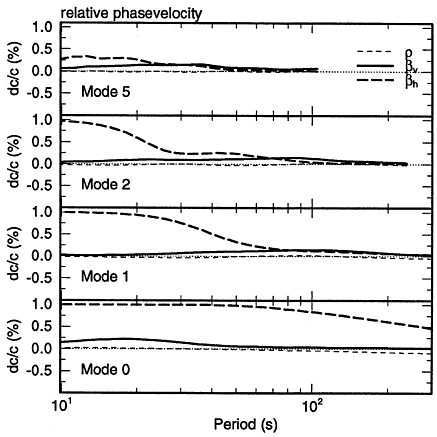

Fig. 4. Relative Love wave phase velocity perturbations dc/c as function of period for a 1% increase in each model parameter in the upper 220 km.

it is difficult to resolve these parameters from Love wave data.

4. Experiment

In this experiment we investigate the bias in the inversion of Rayleigh phase velocities using the alternative parameter set. Phase velocity perturbations have been computed for an input model that has the following variations in the medium parameters with respect to PREM: $\delta\beta_V/\beta_V = 5\%$, $\delta\alpha_V/\alpha_V = 2.5\%$, $\delta\alpha_H/\alpha_H = 2.5\%$, $\delta\gamma/\gamma = 5\%$, $\delta\rho/\rho = 5\%$. These model parameters are only perturbed in the upper 220 km. Phase velocity perturbations have been computed at 10 frequencies between the periods of 20 and 200 s for the fundamental mode up to the fifth higher mode. The data-set is inverted for the reduced parameters (β_V, α_+, α_-). The input model is projected on these parameters using the relations $\alpha_+ = 1/2(\alpha_H + \alpha_V) - \gamma$ and $\alpha_- = 1/2(\alpha_H - \alpha_V)$. Each parameter is defined over 5 layers.

The inversion is carried out using the algorithm of Tarantola and Valette (1982). The data is assumed to be uncorrelated and thus the covariance matrix for the data C_D has only diagonal terms which represent the variance. A realistic standard error of 40 m/s has been assigned to the phase velocity data (Nakanishi and Anderson, 1984; Van Heijst et al., 1994). For the a priori covariance matrix of the model parameters C_m we only take the diagonal elements as non-zero by which C_m acts as norm damping on the inversion. Using a trade-off curve between data misfit and model an optimum solution has been determined which is displayed in Fig. 5.

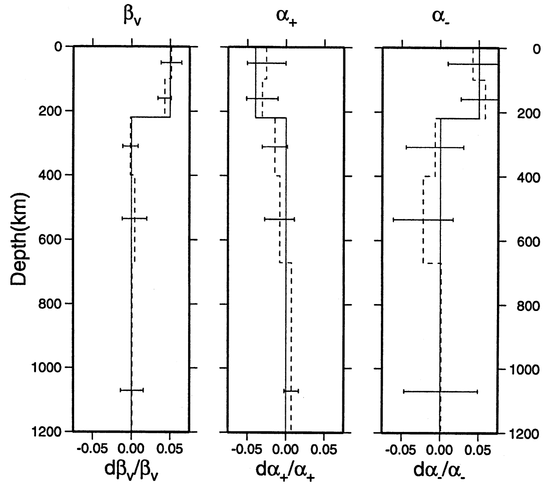

Fig. 5. Synthetic test of the inversion of phase velocity data. Solid line indicates the input model. The dashed line represents the model obtained by inversion. (a) Inversion results for $[\partial\beta_V/\beta_V]$. (b) Inversion results for $[\partial\alpha_+/\alpha_+]$. (c) Results for $[d\alpha_-/\alpha_-]$.

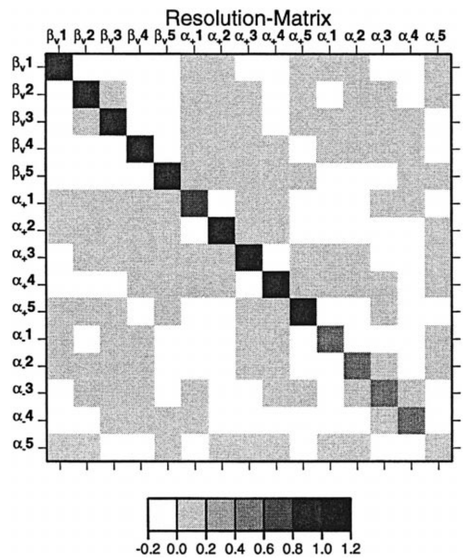

Fig. 6. Resolution matrix for inversion shown in Fig. 5. The parameters increase for $\beta_V 1$ which corresponds to the top layer of β_V to $\alpha_- 5$ the deepest layer of α_-. Note that the zero contour separates the white region from the light-gray region.

We see that the inversion reveals the parameters up to a high degree and well within the uncertainties. The best determined parameter, which has the smallest variances is β_V followed by α_+. The poorest determined parameter is α_- and the increase of variance with depth for the parameter reflects the poor depth sensitivity to this parameter. The resolution matrix shows that β_V and α_+ are well resolved (see Fig. 6). α_- is much poorer resolved, which can be expected from the little depth penetration. The resolution matrix also shows that are little trade-offs between the parameters.

5. Discussion

We have shown that for periods $20 < T < 200$ s the inversion of Rayleigh waves in a transverse isotropic medium has some inherit trade-offs. By introducing a new parameter set we have shown that one can only resolve three parameters (β_V, α_+, α_-). For Love waves the modelling results show that only β_H can be resolved. However, this parameter set should be used with some caution. First, we have shown that the density and γ_- are difficult to resolve but this does not mean that they are completely negligible. When additional information about these parameters is available this should be included in the inversion. Second, Love wave data alone cannot resolve β_V but this does not mean its influence is negligible. When a joint inversion of both Love and Rayleigh waves is carried out the shift in the Love phase velocity due to β_V can be taken into account as it is constrained by the Rayleigh wave. Thirdly, the partial derivative for β_V should not be used for an isotropic inversion of Rayleigh wave data in which case Anderson's partial derivative for β_V should be used. Finally, it is important to realise that it is very important to include the higher modes in the data-set. The differences in the higher mode partial derivatives make it possible to resolve α_+ and α_-. From fundamental mode data alone one can probably only resolve β_V.

The proposed parameterisation is particular useful when the Earth is modelled by transverse isotropy such as in regional studies of phase velocity data (e.g., Nishimura and Forsyth, 1989) or waveform inversion (see Nolet, 1990). It is interesting to note that in the proposed parameterisation, the Rayleigh wave partial derivative for $[\partial c/\partial\beta_V]$ has a shape very similar to the Love wave partial derivative for $[\partial c/\partial\beta_H]$.

This parameterisation can also be used for a more general anisotropic medium. Maupin (1985) has shown that for a given source–receiver path, the partial derivatives of the general anisotropic medium can be written as linear combination of those for the transverse isotropic medium. Instead of using the partial derivatives for the Anderson parameter set one can use our proposed parameter set which has less trade-offs.

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