EXPERIMENTAL TESTING OF TRUNCATED RAMAN-NATH SOLUTIONS

A. DEFEBVRE
Faculté Libre des Sciences
13, rue de Toul
F-59046 Lille CEDEX, France

R.A. MERTENS
Instituut voor Theoretische Mechanica
Rijksuniversiteit Gent
Krijgsalaan 281
B-9000 Gent, Belgium

J.-P. OTTOY
Seminarie Toegepaste Wiskunde en Biometrie
Rijksuniversiteit Gent
Coupure Links 653
B-9000 Gent, Belgium

W. HEREMAN
Department of Mathematics
The University of Wisconsin
Madison, WI 53706
USA

Proceedings ULTRASONICS INTERNATIONAL 87, pp. 78-83 (1987)
EXPERIMENTAL TESTING OF TRUNCATED RAMAN-NATH SYSTEM SOLUTIONS

A. Defebvre *, R.A. Mertens **, J.P. Ottoy †, W. Hereman **

* Faculté Libre des Sciences, 13, rue de Toul, P-59046 Lille Cedex, France
** Institut voor Theoretische Mechanica, Rijksuniversiteit Gent, Krijgslaan 281, B-9000 Gent, Belgium
† Seminarie Toegepaste Wiskunde en Biometrie, Rijksuniversiteit Gent, Coupure Links 653, B-9000 Gent, Belgium
** Department of Mathematics, The University of Wisconsin, Madison, WI 53706, USA

Theoretical results of light diffraction by ultrasonic waves are derived from the generalised theory of Raman and Nath solved successively by either Laplace-Transform method or NOA method treated by both eigenvalue and Heaviside's operational methods. These theories are summarized and their results are compared to both those of previous more descriptive theories and experiments.

INTRODUCTION

Diffraction of light by ultrasonic waves has been studied for a long time and the solutions proceed from either phenomenological or global methods. In the first way Defebvre [1] proposed a geometrical and diffactive theory where curvature of the light rays inside the ultrasonic beams and diffraction alone at the exit plane were involved; as to Hargrove [2] the progressive diffraction in the medium was considered.

In the second way a lot of solutions were derived from the generalised theory of Raman and Nath [3] and here we can recall Leroy's work [4] using the Laplace Transform method which greatly improved an approximate formula obtained by the generating function solution [5] and finally the Nth order approximation (NOA) method from Mertens et al [6].

The aim of this paper is to compare the results of the latter theoretical solution (NOA) to the previous ones and to check them by experiments using high frequency ultrasonic waves having large amplitude and beamwidth.

I. SURVEY OF THE THEORIES

We summarize some recent methods of solution of a Nth order truncated system of Raman-Nath equations [3]:

\[ 2 \frac{d\varphi_p}{d\xi} - (\varphi_{p-1} - \varphi_{p+1}) = ip^2 \rho \varphi_p, \quad p = 0, 1, \ldots, N (\varphi_{N+1} \approx 0) \]  \hspace{1cm} (1)

where \( \varphi_p \) is the amplitude of the diffracted light-wave of order \( p \), with

\[ \xi = \frac{2\pi z}{\lambda}, \quad v = \frac{\xi}{v}, \quad \rho = \frac{\lambda^2}{n_0\Lambda^2 \Delta n}, \quad \varphi_p(0) = \delta_{n0} \]  \hspace{1cm} (2)

\( \lambda \) and \( \Lambda \) : light and ultrasonic wavelengths respectively; \( L \) : ultrasonic beamwidth;
Δn : peak modulation index ; δ_{n0} : Kronecker's symbol. Three successive solutions are considered.

1) The modified "Laplace Transform" method [4,5]

The solution of the truncated system [1] is assumed to be convenient for the complete one and is written as

\[ \varphi_p(\xi) = \beta^P \sum_{t=0}^{\infty} a_t^p J_{p,t}(\beta \xi) \]  

(3)

J_n(z) being the Bessel function of order n, \( \beta \) (with \( 1 < \beta < 3 \)) an accelerating convergence coefficient ; the \( a_t^p \) are derived from recurrent relations and expressed as numerical coefficients independent from \( \rho \) and \( \xi \).

2) The eigenvalue method [6]

The intensities of the successive diffraction orders are given by

\[ I_n(v) = \delta_{n0} - 4(1 + \delta_{n0}) \sum_{j,k=1}^{N+1} c_j c_k a_{j,k}^{(n)} \frac{\sin^2(s_j - s_k)}{s_j - s_k} \frac{v^2}{4}, \]  

(4)

where the \( s_k \) are the real eigenvalues of a Hermitian matrix, \( a_{j,k}^{(n)} \) the corresponding eigenvectors \( (a_0^{(n)} = \sqrt{2}/2) \) and \( c_k \) real constants obtained from a linear equation system.

3) The Heaviside operational method [6]

It gives equivalent solutions to those obtained by the previous one.

More details about those methods as well as phenomenological approaches are available in given references.

II. COMPARISON OF THE THEORETICAL RESULTS MUTUALLY AND WITH THE EXPERIMENTAL DATA

The theoretical fringe intensities \( I_p \) vs Δn in Leroy Laplace Transform theory (L) and NOA-Mertens' one (M) are compared to those of previous phenomenological approaches : Defevre (D) [7] and Hargrove (H) [2] and to the approximative Raman-Nath (N) treatment \( I_n = J_2^N(v) \) up to high diffraction orders. In a second step these results are checked with experimental data of Defevre [1] for ultrasonic frequencies ranging from 1 to 5 MHz, with large beamwidth \( (L = 5 \text{ cm}) \) and Δn values up to \( 19 \times 10^{-6} \) in water \( (n_0 = 1.333) \) and using \( \lambda = 0.54607 \mu\text{m} \). Under those conditions the Raman-Nath parameter \( v \) and the Klein-Cook parameter \( Q = \frac{2\pi AL / n_0^2}{\lambda^2} \) are ranging from 0 to 10 and 0.06 to 1.49 respectively. The parameter \( \xi = \frac{2\pi}{\lambda} z (\Delta n / n_0) \), which gives the extreme path of the light rays inside the ultrasonic beam in geometrical theory, grows up to 3.25.

1) Theoretical curves

Whereas all theories are in very good agreement with the original Raman Nath's solution at 1 MHz, we notice significant deviations from this model when the frequency increases.

At intermediate frequency \( f = 3073 \text{ kHz} \) L and M's theoretical curves can be
superimposed except at the immediate neighbouring of the maxima where L's are slightly higher than M's; however, the latter results are available over larger domains for Δn and p than the former (up to Δn = 19.10⁻⁶ and p < 9, Fig. 1) without any loss of accuracy.

At 5 MHz, although the general features are similar in H, L and M (Fig. 2) we can point out many differences in the light intensity distribution of the different diffraction orders.

- For 0th order, M's values are always smaller than H and L's except for the two minimum values in close coincidence. The same analysis is also convenient for 2d order.
- At 1st order, the starting M's values are smaller than H-D's; for larger Δn, M's are oscillating around H's.
- For 3d order, H and M display a progressive divergence for Δn > 7.10⁻⁶ and a parallel evolution of L's and M's values.
- At higher orders p = 4,5 the M's values lie between H and L.

2. Comparison with experimental data

Spectra are displayed in a reduced coordinate system using both theoretical and experimental values of one order as unity. Vertical full or dashed lines are the theoretical values (respectively D and M on the left-hand; L and H- or H+ on the right-hand); horizontal dotted or full lines: respectively Debevree's real or mean experimental values.

2.1. At 3 MHz, we give (Fig. 3) spectra up to extinction (noted "ext") of the 5th order. We notice the increasing mismatch between N's results and experiments when Δn increases, mainly for the orders on the outside of the fading orders (Fig. 3 c, d, e, f). For the other theories:

- At low level of ultrasonic excitation (ext 0, Fig. 3a), D, M, L completely matched the experiments. After that, for the next case (ext 1, Fig. 3b) equivalent M and L's are better than D's over the whole spectrum.
- At medium level (ext 2, Fig. 3c), M and L are equivalent with a better 6th order in M; D's values are convenient except for the 1st order where there is a large discrepancy compared to experiment; for (ext 3, Fig. 3d) the continuous growth for experimental I_p (p = 0, 2, 4, 5) is fairly interpreted in L and M, completely similar but this last treatment also predicts in a realistic way the outer orders 6, 7 and 8 not available in L.
- At high levels (ext 4, Fig. 3e and ext 5, Fig. 3f) inside orders in M are good but high outside orders appear systematically with intensities larger than the experimental ones; so that in the case of ext 4 D's are pretty convenient (except for the 5th order). Complete comparison with L could not be achieved because we have too few L's values but the latter appear to be interesting.

A sharper analysis up to the second order extinction which corresponds to the entrance in the acoustic surface in the refraction-diffraction Debevree's theory [1, 5, 7] (Δ < 1.672) was also achieved. In that domain the differences between M and L's are quite small, with a slight shift in Δn values and their results are in quite good agreement with our experimental data (displayed at the conference). The spectra show the effective corrections brought to N's values by the other theories; however, beyond ext 1, theoretical values appear systematically smaller than experimental ones for the inner orders.

2.2. At 5 MHz (Fig. 4), we observe a general agreement between theoretical and experimental results with an advantage for the neighbouring M and L results. For those last ones the Δn choice can produce some slight differences at the beginning; at ext 0 (Fig. 4a) it seems better to choose a smaller value for M than L and H; at ext 1 (Fig. 4b) if we fit the first and second orders in M the third has too high a value; alternately for the same choice Δn in M and L (Fig. 4c) with a good fitting for the third order in M, the second one is too high compared to the experiment.

80 Ultrasonics International 87 Conf. Proc.
The next spectra (Fig. 4d, e, f) establish the excellence of L and M's predictions which are superior to the previous approximate D and H's theories. Despite the differences between theoretical values of diffraction orders in L and M above mentioned (Fig. 2), their relative ratios lie in the experimentally defined ranges up to the extreme ultrasonic excitation (Fig. 4f).

CONCLUSION

The comparison between phenomenological (D and H) and global explanations (L and M) of Debye-Sears phenomena establishes clearly the leading part of progressive diffraction of light inside the ultrasonic beam mainly under high Q or E values (i.e. large penetration of light in caustic surface region in the geometrical treatment). Global explanation derived from the generalised Raman-Nath's theory is valuable and many successful solutions (Leroy and Mertens here) can be proposed. Their respective advantages have to be found in their peculiar algorithm capacities.

ACKNOWLEDGEMENTS

One of the authors (R.A.M.) wishes to thank the Belgian National Science Foundation for research grants.

REFERENCES


Fig. 1 - Fringe intensities $I_p$ vs $\Delta n$ according to NOA-MERTENS' theory

$f = 3073$ kHz; $Q = 0.54$

Fig. 3 - Theoretical (D, M, N, L) and experimental spectra

(3 MHz; $Q = 0.54$; $1.125 < \xi < 2.30$)
Fig. 4 - Theoretical (M, D, N, L) and experimental spectra
(2 MHz; Q = 1.49; 1.790 < f < 3.243)

a) ext. 0, f = 5095.5 kHz: \( \Delta n_p = 3.73 \times 10^{-6} \), \( \Delta n_p' = 4.05 \times 10^{-4} \), \( \Delta n_p'' = 4.11 \times 10^{-5} \), \( \Delta n_p''' = 4.065 \times 10^{-6} \)

b) ext. 1, f = 5095.5 kHz: \( \Delta n_p = 6.08 \times 10^{-5} \), \( \Delta n_p' = 7.1 \times 10^{-6} \), \( \Delta n_p'' = 7.00 \times 10^{-7} \), \( \Delta n_p''' = 6.9 \times 10^{-6} \)

c) ext. 1, f = 5096.0 kHz: \( \Delta n_p = 6.62 \times 10^{-5} \), \( \Delta n_p' = 7.1 \times 10^{-6} \), \( \Delta n_p'' = 6.60 \times 10^{-7} \), \( \Delta n_p''' = 6.6 \times 10^{-6} \)

d) ext. 2, f = 5095.69 kHz: \( \Delta n_p = 10.56 \times 10^{-5} \), \( \Delta n_p' = 10.85 \times 10^{-6} \), \( \Delta n_p'' = 10.85 \times 10^{-7} \), \( \Delta n_p''' = 10.85 \times 10^{-6} \)

e) ext. 2, f = 5098.46 kHz: \( \Delta n_p = 11.13 \times 10^{-5} \), \( \Delta n_p' = 11.45 \times 10^{-6} \), \( \Delta n_p'' = 11.45 \times 10^{-7} \), \( \Delta n_p''' = 11.45 \times 10^{-6} \)

f) f = 5098 kHz: \( \Delta n_p = 12.25 \times 10^{-5} \), \( \Delta n_p' = 12.6 \times 10^{-5} \), \( \Delta n_p'' = 12.65 \times 10^{-6} \), \( \Delta n_p''' = 12.6 \times 10^{-5} \)

Fig. 2 - Comparison between fringe intensities according to Margoza's theory (M), Laplace transform Leroy's theory (L----) and HOA-Martens' theory
(M) \( f = 5095.5 \text{ kHz}; \ Q = 1.49 \)