Mathematical Modeling – A 10 Step Procedure

1. Draw a picture

2. Identify the mechanisms (i.e., how the process works). Make a list of assumptions.

3. Identify the pertinent variables and properties and specify a nomenclature.

4. Develop the appropriate differential equation(s) by performing mass, energy, and or momentum balances as appropriate and using the constituitive relationships (e.g., Newton’s law of viscosity (relating stress to velocity), Fourier’s law of heat conduction (relating heat flux to temperature), Fick’s law of diffusion (relating species mass flux to concentration)). Add new assumptions to the list of assumptions.

5. Identify restricting conditions (i.e. boundary and initial conditions) required to solve the differential equation(s). Add new assumptions to the list of assumptions.

6. Non-dimensionalize the independent and dependent variables of the problem if possible.

7. Write down what you expect the solution to look like (e.g., draw a graph).

8. Solve the problem if possible. If too complicated (or not solvable with the methods at hand), simplify and solve. Add all simplifying assumptions to the list of assumptions.

9. Check the solution to confirm that it gives sensible results. Confirm that:
   - the solution satisfies the differential equation
   - the solution goes to the correct result at the boundaries and in limiting cases of time (e.g., at \( t = 0 \), short time, long time and \( t \rightarrow \infty \))
   - a plot of the solution is sensible

10. How does the solution behave compared with what you expected? If it is different, does the solution make sense? If not (e.g., it predicts negative numbers when only positive numbers are possible), then re-examine what you did in Steps 2-9.