Fig. 4. Variation of the function $f(t)$ defined in (10). Crosses are the results of empirical formula given in (12)-(14).

Fig. 5. Equi-error contour in the complex plane.

Fig. 6. Maliuzhinets' function for real arguments.

REFERENCES


Diffraction by a Wide Double Wedge with Cylindrically Capped Edges

A. Z. ELSHERBENI, STUDENT MEMBER, IEEE, AND MICHAEL HAMID, FELLOW, IEEE

Abstract—The diffraction by two conducting sharp wedges with cylindrically capped edges is investigated using a recent asymptotic

Manuscript received July 21, 1985; revised September 13, 1985. This work was supported by the Natural Sciences and Engineering Research Council of Canada and by the University of Manitoba Faculty of Graduate Studies.

The authors are with the Antenna Laboratory, Department of Electrical Engineering, University of Manitoba, Winnipeg, Canada, R3T 2N2.

IEEE Log Number 8608844.
solution proposed by the authors for the diffraction by a wide double wedge. An E-polarized plane wave incident at any angle is considered and the cap is assumed to be either a conducting or dielectric cylinder whose axis coincides with the wedge edge and its radius is much less than the separation between the two virtual sharp edges. The effects of the cap radius, permittivity, and wedge angle on the diffraction pattern, transmission coefficient, and edge-edge interaction term are presented. The transmission coefficient of the aperture is increased over the uncapped wedge case for dielectric caps and decreased for conducting caps. Other effects of the caps on the diffraction pattern such as beamwidth, level, and position of the first sidelobe are also investigated.

INTRODUCTION

The double sharp wedge diffraction problem has been solved asymptotically by the authors [1] for an incident E-polarized plane wave. Following this treatment the effect of edge rounding of the two wedges on the radiation pattern and the transmission coefficient has been also presented [2]. It is found that the edge rounding decreases the transmission coefficient of the double rounded wedge. In this article, we investigate the effect of loading the sharp edges of the two wedges by a cylindrical cap whose axis coincides with the edge. The cap is considered to be a conducting or dielectric infinite cylinder with circular cross section.

The diffraction of an incident E-polarized plane wave by a dielectric capped wedge was derived by Adely [3] and generalized by Towaj et al. [4] to many concentric dielectric shells. Hamid [5] presented a diffraction coefficient for the dielectrically capped wedge, whereas Hamid and Towaj [6] showed the effects of the dielectric cap on the radiation characteristics of a capped half-plane excited by a line source field. The diffraction by a conducting capped wedge was solved exactly by Karp [7] in terms of an infinite series of angular eigenfunctions. The conducting cap half-plane was studied by many authors. Keller [8] investigated the darkness of the shadow of a rounded and a caged screen using ray theory and the geometrical theory of diffraction (GTD). A modification to Keller's approach for this problem was proposed by Kouyoumjian and Burnside [9] by including additional ray systems and using more accurate diffraction coefficient. Keller and Magiros [10] gave an alternative exact solution for the diffraction by a capped half-plane in terms of an infinite series of radial eigenfunctions and showed that their solution agrees with Karp's solution. They also derived an approximate asymptotic expression for the diffraction field in the shadow and lighted regions. Starting with the exact solution, Chu et al. [11] derived an alternative uniform far-field asymptotic solution for the diffraction of an electromagnetic plane wave by a conducting capped half-plane.

The present analysis of multiple scattering between the two capped wedges is based upon the exact field expressions for a single capped wedge due to an incident plane wave at any angle, the line source field derived using the boundary value approach [5], [7], as well as the technique proposed by Karp and Russek [12] for the diffraction by a wide slit. The dependence of the diffraction field on the interaction between the two capped wedges is clearly represented by simple relations.

FORMULATION

Using the circular cylindrical coordinates (ρ, φ, z), the single capped wedge geometry may be defined by two half-planes φ = γ and φ = 2π − γ intersecting along the z axis and an infinite circular cylinder of radius r whose axis coincides with the z axis. We consider the E-polarization case (transverse magnetic (TM) with respect to the z axis) where the electric field has a z component only with all vectors independent of z, while the time dependence e^{jωt} is considered and suppressed throughout.

Employing the Karp and Russek technique for the interaction between the two capped wedges, the diffracted field due to an incident plane wave plus a fictitious line source, located in the same position as the edge of the second wedge, is derived.

For a plane wave of unit amplitude incident at angle φ₀ with respect to the negative x axis, the incident field E₀^i is given by

$$ E₀^i = e^{jkr \cos (φ - φ₀) } $$

where k is the wavenumber 2π/λ and λ is the wavelength. While for a line source of unit amplitude at (ρ₀, φ₀) and parallel to the z axis, the incident field E₁^i is given by

$$ E₁^i = \frac{π}{2j} H₀(kR) $$

where R is the distance between the line source and the field point, and H₀(x) is the Hankel function of the second kind of order zero and argument x. For all Hankel functions the superscript (2) is implied throughout.

The total field in the presence of a wedge is the geometrical optics field (E₀) plus the diffracted field. The diffracted fields due to a plane wave and a line source field incident on a conducting sharp wedge are given by E₀^d and E₁^d, respectively, where

$$ E₀^d = \frac{π}{2j} H₀(kρ₀) g^d(φ, φ₀, ν) $$

$$ E₁^d = \frac{π}{2j} H₀(kρ₀) f^d(φ, φ₀, ν) $$

with

$$ g^d(φ, φ₀, ν) = \frac{\sin (π/ν)}{πν} \left[ \cos \left( \frac{π}{ν} \right) - \cos \left( φ - φ₀ \right) \right]^{-1} $$

$$ f^d(φ, φ₀, ν) = \frac{π}{2j} H₀(kρ₀) g^d(φ, φ₀, ν) $$

while the superscript S refers to a sharp wedge.

For conducting capped wedge the corresponding diffracted field patterns g^c and f^c due to a plane wave and a line source field are given for small values of kr by

$$ g^c(φ, φ₀, ν, r) = g^d(φ, φ₀, ν) $$

$$ f^c(φ, φ₀, ν, r) = f^d(φ, φ₀, ν) $$

respectively, while the superscript C refers to a conducting capped wedge.
However, for a dielectric capped wedge the diffracted field patterns due to a plane wave and a line source field are \(g_{\text{D}}(\phi, \phi_0, \nu, \gamma, \epsilon)\) and \(f_{\text{D}}(\phi, \phi_0, \nu)\), respectively, and given for \(k_{lr} \) not too large relative to unity by

\[
g_{\text{D}}(\phi, \phi_0, \nu, \gamma, \epsilon) = g_{\text{D}}(\phi, \phi_0, \nu) = \sum_{n=-1}^{\infty} j^{n+1}T_{n/\nu} \sin \frac{n}{\nu} \left( \phi - \gamma \right) \sin \frac{n}{\nu} \left( \phi_0 - \gamma \right) (9)
\]

\[
f_{\text{D}}(\phi, \phi_0, \nu, \gamma, \epsilon) = f_{\text{D}}(\phi, \phi_0, \nu) = -\sum_{n=-1}^{\infty} j^{n+1}T_{n/\nu} \sin \frac{n}{\nu} \left( \phi - \gamma \right) \sin \frac{n}{\nu} \left( \phi_0 - \gamma \right) (10)
\]

where the superscript \(D\) refers to a dielectric capped wedge and

\[
T_{n/\nu} = k_J^{n/\nu}(k_1r)J_{n/\nu}^{'}(kr) - k_J^{n/\nu}(kr)J_{n/\nu}^{'}(k_1r)

+ k_J^{n/\nu}(k_1r)H_{n/\nu}^{'}(kr) - k_J^{n/\nu}(kr)H_{n/\nu}^{'}(k_1r)
\]

while \(k_1 = \sqrt{\epsilon_1 k_0}\), \(\epsilon_1\) is the relative permittivity of the dielectric cap, and the prime indicates differentiation with respect to the total argument.

For small values of \(kr\) and \(k_{lr}\) relative to unity, the geometrical optics component due to the cylindrical cap (in the presence of the wedge) is much smaller than the diffracted field component [5]. Therefore, second term (which accounts for the reflected and diffracted fields due to a cylindrical cap) in each of (7)–(10) may be considered as a perturbation to the diffracted field due to a sharp wedge.

From (3)–(10) one notices that the field diffracted by a sharp wedge, a conducting capped wedge or a dielectric capped wedge, due to either a plane wave or a line source excitation, has the appearance of a cylindrical wave emanating from a fictitious line source at the edge of the wedge.

For the case of two capped wedges separated by \(s_{12}\) (sharp edge to edge distance where \(ks_{12} \gg 1\)) and illuminated by a plane wave of unit amplitude (see Fig. 1), the total diffracted field \(E^d\) is given for \(-\pi/2 + 2\alpha \leq \theta \leq \pi/2 - 2\beta\) and \(0 \leq \alpha \leq \beta < \pi/4\) by

\[
E^d = E^d_1 + E^d_2
\]

where

\[
E^d_1 = \frac{\pi}{2j} H_0(kp_1)\left[ e^{-jk_1s_12} s_{01} g(\phi_1, \phi_0, \nu_1, \gamma_1, \epsilon_1) + c_1 f(\phi_1, s_{12}, \pi, \nu_1, r_1, \epsilon_1) \right]
\]

\[
E^d_2 = \frac{\pi}{2j} H_0(kp_2)\left[ e^{+jk_2s_12} s_{02} g(\phi_2, \phi_0, \nu_2, \gamma_2, \epsilon_2) + c_2 f(\phi_2, s_{12}, \pi, \nu_2, r_2, \epsilon_2) \right]
\]

\[
s_{12} = s_1 + s_2
\]

\[
c_1 \text{ and } c_2 \text{ are the unknown strengths of the fictitious line sources at the edges of wedge } B \text{ and wedge } A, \text{ respectively. Following the convention that angles measured clockwise are negative and referring to Fig. 1, we have, for any far-field point, } r_1 = p - s_1, \text{ sin } \theta_1, r_2 = p + s_2, \text{ sin } \theta_1, \phi_1 = (\pi/2), \theta_1, \phi_2 = (\pi/2) - \theta_1, \phi_0 = (\pi/2) + \theta_0, \text{ and } \phi_0 = (\pi/2) - \theta_0.
\]

For the determination of \(c_1\) and \(c_2\), one may follow the same analysis given in [12] by imposing the requirement that the diffracted fields by the two wedges be consistent with one another. This leads to the following:

\[
c_1 = \left[ e^{-jk_1s_12} s_{01} g(\pi, \phi_0, \nu_1, r_1, \gamma_1, \epsilon_1) + e^{+jk_2s_12} s_{02} g(\pi, \phi_0, \nu_2, r_2, \gamma_2, \epsilon_2) \right] / w
\]

\[
c_2 = \left[ e^{+jk_1s_12} s_{01} g(\pi, \phi_0, \nu_1, r_1, \gamma_1, \epsilon_1) + e^{-jk_2s_12} s_{02} g(\pi, \phi_0, \nu_2, r_2, \gamma_2, \epsilon_2) \right] / w
\]

where

\[
w = 1 - f(\pi, s_{12}, \pi, \nu_1, r_1, \gamma_1) f(\pi, s_{12}, \pi, \nu_2, r_2, \gamma_2)
\]

Hence, the total diffracted field \(E^d\) is fully determined and can be written in the following form:

\[
E^d = \frac{e^{-jk_0s}}{\sqrt{\pi kp_0}} F(\theta)
\]

where \(F(\theta)\) is the diffraction pattern of the double capped wedge.

Another convenient representation for the total field can be written as

\[
E^t = E' + E''
\]

The noninteraction term \(E'\) represents the excitation of each capped wedge separately by the incident plane wave. This may be a good approximation for extremely wide wedges. However, for narrower wedges the interaction term \(E''\) is required.

The transmission coefficient \(T\) for a plane wave incident at angle \(\theta_0\) is given by

\[
T = \text{Re} \left\{ (1 - jF) / k s_{12} \right\}
\]

where

\[
F = \lim_{\theta \to 0} F(\theta).
\]

**DISCUSSION OF NUMERICAL RESULTS**

Although the formulation is general, it seems reasonable to concentrate in our discussion on the symmetric configuration where \(\theta_0 = 0^\circ, \alpha = \beta = \gamma_1 = r_1 = r_2 = r, \epsilon_1 = \epsilon_2 = \epsilon_r\) and \(s_1 = s_2 = s\). Fig. 2 shows the effect of the conducting cap radius on the normalized \(E^\theta\) pattern of a double capped wedge for \(ks = 8\) and \(\gamma = 5^\circ\). It is found that the beamwidth increases with the cap radius. For example, as \(kr\) assume the values 0.0, 0.3, 0.6, and 0.9 the beamwidth values are 17.03°, 18.92°, 21.5° and 22.59°, respectively. However, for the case of a dielectric capped double wedge, Fig. 3 shows that the beamwidth decreases with \(\epsilon_r\) for \(ks = 7, \gamma = 10^\circ\) and \(kr = 0.5\). For the indicated values of \(\epsilon_r\), namely, 1, 5, 7 and 9, the corresponding values of the beamwidth are 21.96°, 19.11°, 17.72° and 16.82°, respectively. Similar reduction in the beamwidth is observed when the dielectric caps electric radius \(k_{lr}\) increases although this is not shown here.
The dependence of $T$ on $kr$ is shown in Fig. 4 for a conducting capped slit. It is clear that $T$ decreases in general with $kr$. However, for large values of $ks$ the effect of capping decreases and $T$ approaches unity. For a dielectrically capped slit, $T$ is found to be increasing with $\epsilon_r$ as shown in Fig. 5 for $kr = 0.5$. The agreement between the decrease in $T$ with increasing $kr$ as shown in Fig. 4 as well as the increasing beamwidth of the radiation pattern with $kr$ as shown in Fig. 2 establishes the relation between $T$ and the beamwidth. In other words $T$ decreases with increasing beamwidth, as expected.

In Fig. 6, the effect of the interior wedge angle $2\gamma$ on $T$ is shown for $kr = 0.5$ and $\epsilon_r = 4$. It is found that the peak-to-peak value of the oscillations of $T$ increases with $\gamma$ for a fixed values for $kr$ and $\epsilon_r$. Similar effect is also observed for a conducting capped double wedge. For large $ks$ the oscillations die down and $T$ converges to unity as expected.

The possibility of increasing or decreasing $T$ over a certain range of $ks$ by loading the aperture of a wide double sharp wedge by a cylindrical dielectric shell with radial and azimuthal permittivity profiles [13] is presently being investigated following the solution of the diffraction by a double wedge and a parallel cylinder [14]. The results of this approach will be reported in a future article.
Fig. 4. Transmission coefficient versus $ks$ of a conducting capped slit.

Fig. 5. Transmission coefficient versus $ks$ of a dielectric capped slit for $kr = 0.5$.

CONCLUSION

Our previous solution for the diffraction of an incident plane wave by a wide double wedge has been extended to include the effect of capping the edges by small cylindrical dielectric or conducting caps. The present method is applicable to symmetric or asymmetric geometry. The plane wave may be incident at any angle, and the generalization for a line source excitation is simple and straightforward. The results show an increase in the transmission coefficient for dielectric caps whereas the conducting caps yield a lower transmission coefficient relative to the sharp edge case. Edge capping produces further variations in the radiation characteristics such as beamwidth, level, and position of the first sidelobe.

REFERENCES