Circular Sectoral Waveguides

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Abstract

Hollow cylindrical waveguides, which have pie-shaped cross sections, are described. The explicit equations for the field components of the TE and TM waves are given, and the modal patterns for a number of modes are plotted. The relation between the field distribution inside circular and sectoral waveguides is discussed. It is shown that the electric and magnetic fields of the modes TE_{1n} and TM_{1n} display the edge singularity, when the sectoral angle is greater than π. Formulas convenient for the evaluation of Bessel functions of fractional order on a personal computer are also given.

1. Introduction

Hollow waveguides of rectangular and circular cross section are discussed in many elementary electromagnetic-theory textbooks. As is well known, the electromagnetic field inside these waveguides can be described in terms of simple functions, like trigonometric and/or Bessel functions. The mode notation in rectangular and circular waveguides has been standardized in [1], and the most comprehensive collection of modal patterns in rectangular and circular waveguides can be found in [2].

Another waveguide cross-section, for which the inside field components can also be described by relatively simple mathematical expressions, is shown in Figure 1. Some advanced electromagnetic theory textbooks [3-5] briefly mention the existence of such sectoral waveguides (also called circular-wedge waveguides), but the discussion of individual modes is not readily available.

The present paper intends to give a tutorial presentation of the modes in circular-sectoral waveguides, including plots of the modal fields, and tables of eigenvalues (propagation constants) and cutoff frequencies. The paper also discusses the singularity of either the electric or magnetic field in the vicinity of the metal apex of the wedge. It will be shown that some modes possess this edge singularity, and others don't.

Modal field patterns in this paper have been generated by the personal computer software WGVMAP [6], recently developed for the Computer Applications in Electromagnetic Education center (CAEME). The software package WGVMAP has been developed to interactively use a personal computer for computing and plotting the electric- and magnetic-field lines for both TM and TE modes, propagating inside various types of cylindrical waveguides.

The user can observe the effect of changing any of the physical or electrical parameters, on the resulting electric and magnetic field distribution in a plane perpendicular to the propagation direction. The governing equations are based on the classical separation of variables technique [5], while plotting the field lines is accomplished by previously-developed software for vector field mapping (VMAP) [7-9].

2. Analytic Solution

This section presents the basic equations for the propagating waves inside a circular-sectoral, or wedged, waveguide. The axis of the waveguide is parallel with the z direction, which is also the direction of wave propagation. The waveguide walls are considered to have infinite conductivity, while the medium inside the waveguide is filled with a homogeneous, isotropic, and lossless dielectric material, characterized by the permittivity ε and the permeability μ. For this type of waveguide, the propagating waves can be divided into two basic sets of modes. For one set of these modes, the magnetic field component along the propagation direction is equal to zero. Thus, the resulting waves are called transverse magnetic waves (TM). For the other set of modes, the electric field has a zero component along the propagation direction: therefore, the waves are referred to as transverse electric field waves (TE) [1].

The expressions for the field components for both TM and TE modes can be obtained by solving the Helmholtz wave equation inside the waveguide, using the separation of variables technique [3-5]. The cylindrical coordinate system (r,φ,z) is used, and the resulting field components are given here for a waveguide of radius a and sectoral angle φ₀, as shown in Figure 1. For TE modes, propagating in the positive z direction, the field components are given by

\[
E_{r}(r,\phi,z) = A_{mn} \frac{k_{n}}{\sqrt{\rho}} J_{m}(k_{n} r) \sin(m \phi) e^{-j \beta_{z} z}
\]

(1)

\[
E_{\phi}(r,\phi,z) = A_{mn} \frac{k_{n}}{\sqrt{\rho}} J_{m}(k_{n} r) \cos(m \phi) e^{-j \beta_{z} z}
\]

(2)

\[
E_{z}(r,\phi,z) = 0
\]

(3)

\[
H_{r}(r,\phi,z) = -A_{mn} \frac{k_{n} z}{\sqrt{\rho}} J_{m}(k_{n} r) \sin(m \phi) e^{-j \beta_{z} z}
\]

(4)

\[
H_{\phi}(r,\phi,z) = A_{mn} \frac{mk_{n} z}{\sqrt{\rho}} J_{m}(k_{n} r) \cos(m \phi) e^{-j \beta_{z} z}
\]

(5)

\[
H_{z}(r,\phi,z) = -jA_{mn} \frac{k_{n} z}{\sqrt{\rho}} J_{m}(k_{n} r) \cos(m \phi) e^{-j \beta_{z} z}
\]

(6)

while for TM modes the corresponding field components are

\[
E_{r}(r,\phi,z) = -B_{mn} \frac{k_{n} z}{\sqrt{\rho}} J_{m}(k_{n} r) \sin(m \phi) e^{-j \beta_{z} z}
\]

(7)

\[ E_p(\rho, \phi, z) = -B_p \frac{mk_p}{\sqrt{\mu\varepsilon}} J_m(k_p \rho) \cos(m\phi) e^{-jk_z z} \]  
(8)

\[ H_p(\rho, \phi, z) = B_p \frac{m}{k_p^2} J_m(k_p \rho) \cos(m\phi) e^{-jk_z z} \]  
(9)

\[ E_\phi(\rho, \phi, z) = -jB_p \frac{k^2}{\sqrt{\mu\varepsilon}} J_m(k_p \rho) \sin(m\phi) e^{-jk_z z} \]  
(10)

\[ H_\phi(\rho, \phi, z) = -B_p \frac{k_p}{\mu} J_m(k_p \rho) \sin(m\phi) e^{-jk_z z} \]  
(11)

\[ H_z(\rho, \phi, z) = 0 \]  
(12)

The \( A_{pn} \) and \( B_{pn} \) are the amplitudes of the mode designated by the integer subscripts \( p \) and \( n \), and \( m \) is given by an irrational number

\[ m = p \pi / \phi_0 \]  
(13)

The integer, \( p \), is defined as \( p=0, 1, 2, 3, ... \) for TE modes, and \( p=1, 2, 3, ... \) for TM modes. The wave number, \( k \), is given by

\[ k = \omega / \sqrt{\mu \varepsilon} \]  
(14)

where \( \omega \) is the angular frequency, and

\[ k^2 = k_p^2 - k_\phi^2 \]  
(15)

\[
\begin{align*}
\chi_{mn} & = \chi_{mn} \text{ for } TE_{pn} \text{ modes} \\
\chi_{mn} & = a \text{ for } TM_{pn} \text{ modes}
\end{align*}
\]  
(16)

The \( \chi_{mn} \) are the zeros of the derivative of the Bessel function \( Y_m(\chi') \) and of the Bessel function \( J_m(\chi) \), respectively. For both TE and TM modes, the second subscript, \( n \), is equal to 1, 2, 3, ..., which represents the order of the zeros of \( Y_m(\chi') \) and of \( J_m(\chi) \). The procedure for computing the Bessel function is detailed in Appendix I. In Appendix II, Tables 1-6 list the zeros for waveguides of sectorial angles equal to 180°, 270° and 360°, for both TE and TM modes of excitation. Since \( m \) is an integer for \( \phi_0=90° \) \((m=2p)\), the zeros for a 90°-sectoral waveguide can also be retrieved from Tables 1 and 2.

The cutoff frequency, \( f_c \), for the \( p_n \) mode, is then given by

\[
\chi_{mn} \begin{cases} \
2\pi\sqrt{\mu\varepsilon} & \text{TE}_{pn} \text{ modes} \\
2\pi\varepsilon & \text{TM}_{pn} \text{ modes}
\end{cases}
\]  
(17)

In Appendix III, Table 7 lists the normalized cutoff frequencies of the first 20 propagating modes, with four different sectoral angles.

For hollow circular waveguides, the standard nomenclature uses subscripts \( m \) and \( n \) to denote the individual modes, like \( TE_{mn} \) and \( TM_{mn} \). For sectoral waveguides, \( m \) is not an integer, and it becomes awkward to use five or more digits to denote the first subscript of the wave. Some authors have used fractions and the letter \( \pi \), but we propose to use the integer, \( p \), as the first subscript, and the integer, \( n \), as the second subscript, thus denoting the modes as \( TE_{pn} \) and \( TM_{pn} \). The integer \( p \) denotes the number of half-period variations of the modal field, when an observer moves at a constant radius from one end wall of the sectoral waveguide to the other end wall. This fact can also be described by saying that \( p \) represents the number of axial planes along which the normal component of the electric vector vanishes for TE modes, while for TM modes \( p \) is the number of vortices of the electric-field lines in the transverse plane. The integer \( n \), however, represents the number of coaxial cylindrical arcs, including the outer part of the waveguide boundary, along which the tangential component of the electric (or the normal component of the magnetic) vector vanishes for the TE (or TM) mode. For the TE case, these circular arcs pass through the vortices of the magnetic-field lines.

### 3. Modal Field Patterns

Let us first have a look at the modes in a sectoral waveguide for and angle of 90 degrees. Solid lines represent the electric-field lines, and dashed lines represent the magnetic-field lines. The field with no angular variation, and one half-period radial variation, is the \( TE_{01} \) mode, shown in Figure 2a. Figure 2b shows the \( TE_{02} \) mode field, with no angular variation and two variations in the radial direction. The more complicated the field pattern is, the higher is its cutoff frequency. For instance, it can be seen from Table 7 in Appendix III that the cutoff frequency of the \( TE_{01} \) mode for \( \phi_0=90° \) is in the second place after the \( TE_{11} \) mode, the \( TE_{02} \) mode is in the sixth place, and the \( TE_{03} \) is in the fourteenth.

The electric-field lines of all the \( TE_{mn} \) modes are circular arcs, starting at one plane wall and ending on the other. The magnetic-field lines appear in Figure 2 as radial lines. However, this is so only in the cross-sectional view. Actually, thinking in terms of the three-dimensional behavior, the magnetic-field lines are curved, because they also have a non-vanishing \( z \) component. For instance, close to the cylindrical wall of the waveguide, the radial component of the magnetic field vanishes, and the field lines are pointing out of the cross section (strong \( H_z \) component). The plot of the magnetic-field lines in Figure 2 is made to stop when the field amplitude drops 10 dB below its maximum value.

![Figure 2: TE modes inside a 90° sectoral waveguide.](image)

The \( TM_{mn} \) field, with no angular variation, cannot exist in a sectoral waveguide. This is in contrast to the behavior of the hollow circular waveguide.
interior electric-field lines disappear (field amplitudes smaller than 10 dB below the peak are not plotted). In the region where they are not plotted, the electric-field lines acquire a strong $z$ component. On the other hand, since this is a transverse-magnetic type of field, the magnetic-field lines have no $z$ component. Furthermore, the magnetic-field lines cannot end perpendicular to the conductor walls; thus, all the magnetic-field lines must form closed loops. In Figure 4, there is one family of loops for the TM$_{11}$ mode, there are two families for the TM$_{21}$ mode, four families for the TM$_{22}$ mode, and six families for the TM$_{23}$ mode, etc.

As expected, the TE-type fields have the electric-field lines confined to the transverse plane of the cross section, so that either the electric-field lines end up perpendicular to a conducting wall, or they form closed loops. The corresponding magnetic-field lines of the TE modes are everywhere perpendicular to the electric-field lines. In some portions of the cross section, the magnetic-field lines disappear, because their transverse component becomes very small. These are the regions where the longitudinal-field component is strong, but we cannot see it in the cross-sectional view.

For the TM fields, the magnetic-field lines are always closed loops. The electric-field lines are everywhere perpendicular to the magnetic-field lines. In some portions of the cross section, the electric-field lines of the TM modes end up perpendicular to the conducting walls; in other portions, they disappear from view, because of a strong longitudinal component.

In Figure 5a, we see the TE$_{01}$ mode for a sectoral waveguide with $\theta_0=270^\circ$. The electric-field lines are circles, and the magnetic-field lines are radial lines, truncated at the place where the

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amplitude drops by 10 dB. The mode TE_{11} is shown in Figure 5b. One can notice the concentration of the electric-field lines at the inner edge, the so-called edge effect. Figure 5c represents the TE_{21} mode, which does not show any signs of the edge effect. Three more examples are shown in Figures 5d to 5f, for modes TE_{31}, TE_{41}, and TE_{12}.

The TM modes for a 270° sectoral waveguide are shown in Figures 6a to 6f. For all of these, the magnetic-field lines are closed loops. For a mode TM_{pmn}, there are \( p \cdot n \) families of loops, where the subscript \( p \) pertains to the azimuthal direction, and the subscript \( n \), to the radial direction. At the center of each loop of the magnetic-field lines there is a vortex of the electric-field lines.

Next, we show some examples of sectoral waveguides for \( \phi_0=180° \). The TE_{01} and TM_{01} modes in the 180°-sectoral guide in Figures 7a and 7b obviously represent one-half of the patterns for the TE_{02} and TM_{11} modes in hollow circular waveguides, shown in Figures 8a and 8b. Similar conclusions can be made for any other mode in the 180°-sectoral waveguide. For \( \phi_0=\pi \), Equation (13) yields \( m=\pm p \), so that, formally, the subscripts of the TE and TM modes for 180°-sectoral waveguides and for hollow circular waveguides are the same. In hollow circular waveguides, \( m \) is the number of full-period variations in the azimuthal direction. In circular sectoral waveguides, \( p \) is the number of half-period variations in the azimuthal direction. Thus, the two patterns are specified by the same mathematical functions and, within the common regions of the total cross section, they are identical to each other.

![Figure 6: TM modes inside a 270° sectoral waveguide.](image)

![Figure 7: Field distribution inside a 180° sectoral waveguide.](image)

![Figure 8: Field distribution inside circular waveguide.](image)

![Figure 9: TE modes inside a baffle waveguide.](image)
hollow circular waveguide, shown in Figure 10b. The four modes shown in Figure 11 have the radial variation index \( n = 2 \).

In general, we see that every second mode, \( \text{TE}_{pm} \), in the baffle waveguide has a modal pattern analogous to the \( \text{TE}_{nm} \) mode in the hollow circular waveguide. The reason can be found in Table 6, where it can be seen that each even-valued integer \( p \) corresponds to an integer value of \( m \). The odd-valued integers \( p \) correspond to non-integer values of \( m \); such modes cannot exist in the hollow circular waveguide.

The transverse-magnetic modes in the baffle waveguide are shown in Figures 12a to 12f. The magnetic-field lines are closed loops, while the electric-field lines either end up perpendicularly on the conducting walls, or form vortices. In the same way as for the \( \text{TE} \) modes in the baffle waveguide, every even value of \( p \) corresponds to an identical modal pattern of the hollow circular waveguide, whereas the odd values of \( p \) result in patterns which are unique to the baffle waveguide.

Figures 13a and 13b show the modes \( \text{TE}_{01} \) and \( \text{TE}_{02} \). The electric-field lines are circles, and the magnetic-field lines are radial straight lines. As mentioned before, the transverse-magnetic modes in sectoral waveguides cannot exist for \( p=0 \).

Figure 10: TE modes inside a circular waveguide.

Figure 11: TE modes inside a baffle waveguide.

Figure 12: TM modes inside a baffle waveguide.

Figure 13: TE modes inside a baffle waveguide.

4. Edge Singularity

When the sectoral angle \( \phi \geq \pi \), some modes display a concentration of field at the sectoral edge, such as can be seen in Figures 5b, 6a, 6e, and 9a. It is a known fact that either the electric or the magnetic field at the edge of a conducting wedge may become singular, tending toward infinite magnitude as the distance from the edge tends to zero [11,12]. In a sectoral waveguide, the edge is located at the radius \( p=0 \). Therefore, to check whether a certain mode has an edge singularity, one has to investigate whether the mode behaves as a negative power of \( p \), for small values of \( p \).

For \( \text{TE}_{pm} \) modes, Equations (1) and (5) contain \( p \) in the denominator. Therefore, whether the components \( E_p \) and \( H_p \) are singular depends on the behavior of the Bessel function \( J_n \) as
\( \rho \rightarrow 0 \). From Equation (20) in Appendix I, one finds that the first term in the expansion of \( I_m(x) \) is

\[
I_m(x) = \frac{x^m}{\Gamma(m+1)}
\]

(18)

Except for a multiplicative constant, the \( \rho \)-dependence of \( E_\rho \) and \( H_\phi \) is thus given by

\[
E_\rho, H_\phi \propto \rho^{m-1}
\]

(19)

The smaller \( m \) is, the more pronounced will be the singularity. The value \( m=0 \) is not acceptable, because Equations (1) and (5) contain \( m \) as a factor, making the whole expression zero. For other modes, \( m \) is an irrational number, specified by the integer \( p \) in accordance with (13). The only value of \( p \) which can cause the singularity is \( p=1 \). For a sectoral angle \( \phi_0=3\pi/2 \), one obtains \( m=0,666 \), and for the haffle waveguide, one obtains \( m=0.5 \). Any value of \( p=1 \) will result in \( m>1 \), and thus a positive power of \( \rho \) in (19).

For the TM_{m1n} modes, the components with possible singularities are \( E_\rho \) and \( H_\phi \), as can be seen from Equations (8) and (10). The functional dependence on \( \rho \) is the same as discussed above. In conclusion, the only modes displaying an edge singularity are TE_{11n} and TM_{11n} for \( n=1,2,3, \) etc. As mentioned above, the prerequisite is that the sectoral angle \( \phi_0 \) be larger than \( \pi \).

5. References


Appendix I: Bessel Function

The computation of the Bessel function of the first kind for real arguments and orders is based on the series definition [13]:

\[
J_m(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{2k+m}}{k! \Gamma(k+1+m)}
\]

(20)

To avoid machine overflow, each term in the series was constructed in the special manner shown below:

\[
\frac{(x/2)^{2k+m}}{k! \Gamma(k+1+m)} = \frac{(x/2)^k}{k!} \frac{(x/2)^k}{k!} \frac{(x/2)^k}{k!} \cdots \frac{(x/2)^k}{k!} \frac{(x/2)^m}{\Gamma(k+1+m)}
\]

(21)

where

\[
\frac{(x/2)^k}{k!} = \frac{x/2 x/2 x/2 x/2 x/2}{k k k-1 k-2 2 1}
\]

(22)

\[
\frac{(x/2)^k}{k!} = \frac{x/2 x/2 x/2 x/2 x/2}{k k k-1 k-2 2 1}
\]

(23)

With this procedure, arguments as high as 20 could be evaluated with 5 digits of accuracy at 0 order. For increased order, values for large arguments are obtained with the same accuracy.

The series expression of the Bessel function requires the Gamma function. Therefore, another routine was developed to evaluate the Gamma function for positive real arguments in the range from 1 to 2. This routine is based on the following polynomial approximation [13]:

\[
\Gamma(x+1) = 1 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots + b_6 x^6 + \varepsilon(x)
\]

where

\[
0 \leq x \leq 1, |\varepsilon(x)| \leq 3 \times 10^{-7}
\]

and

\[
b_1 = -0.577191652, b_2 = 0.988205891, b_3 = -0.897056937,

b_4 = 0.918206857, b_5 = -0.756704078, b_6 = 0.482199394,

b_7 = -0.193527818, b_8 = 0.035868343.
\]
### APPENDIX II

**ZEROS OF \( J_m(x) \) AND \( J'_m(x) \)**

#### Table 1: Zeros of \( J_m(x) \) for \( \phi_0 = 180^\circ \)

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#### Table 2: Zeros of \( J'_m(x') \) for \( \phi_0 = 180^\circ \)

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### APPENDIX III

#### Table 5: Zeros of \( J_m(x) \) for \( \phi_0 = 360^\circ \)

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#### Table 6: Zeros of \( J'_m(x') \) for \( \phi_0 = 360^\circ \)

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#### Table 7: Normalized cutoff frequencies.

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<tr>
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\( \phi_0 = 270^\circ \) \hspace{1cm} \( \phi_0 = 360^\circ \)

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### Introducing Feature Article Authors

**Atef Elsherbeni** was born in Cairo, Egypt on January 8, 1954. He received an honor BSc degree in Electronics and Communications, an honor BSc degree in Applied Physics, and a MEng degree in Electrical Engineering, all from Cairo University, Cairo, Egypt, in 1976, 1979, and 1982, respectively, and a PhD degree in Electrical Engineering from University of Waterloo, Winnipeg, Manitoba, Canada, in 1987. He was a Research Assistant with the Faculty of Engineering at Cairo University from 1976 to 1982, and from 1983 to 1986 at the Electrical Engineering Department, Manitoba University, and from January to August, 1987, as a Post-Doctoral Fellow in the same department. He joined the faculty at the University of Mississippi in August, 1987, where he is currently an Associate Professor of Electrical Engineering. His professional interests include scattering and diffraction of electromagnetic waves, numerical techniques, antennas and computer-aided design. He has authored and/or co-authored over 60 technical papers and reports on applied electromagnetics, antenna design, and microwave subjects. Dr. Elsherbeni is a senior member of the IEEE, belonging to the Antennas and Propagation and the Microwave Theory and Techniques Societies. His honorary memberships include the Electromagnetics Academy and the Sigma Xi Scientific Society.

**Darko Kajfez** is a Professor of electrical engineering at the University of Mississippi. His teaching and research interests are in numerical solutions of electromagnetic problems in microwave circuits and antennas. He obtained the PhD degree from the University of California, Berkeley, in 1967, and the electrical engineer's degree (Dipl. Ing.) from the University of Ljubljana, Yugoslavia, in 1953. He co-edited the book *Dielectric Resonators*, and authored three volumes of the graduate textbook *Notes on Microwave Circuits*.

**Sheng Zeng** was born in Hubei, China, on April 28, 1964. She obtained the BS degree in Optical Physics from the University of Optics and Fine Mechanics, Changchun, China, in 1985. She then worked as an Assistant Engineer for China Electronics Import and Export Corporation for four years. She joined the Department of Electrical Engineering at the University of Mississippi in January, 1990, where she is currently a Research Assistant, working towards her MS degree. Her research interests include CAD, numerical methods, and wave propagation.