Analysis of Multilayer Periodic Structures with Different Periodicities using a Hybrid FDTD/GSM Method

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Abstract — An efficient algorithm to compute the scattering properties of multilayered periodic structure with different periodicities using a hybrid finite-difference time-domain/generalized scattering matrix (FDTD/GSM) technique is described. In this algorithm the constant horizontal wavenumber approach is used to compute the scattering parameters of each layer. A Floquet harmonic analysis of the periodic structure is presented, where propagation and evanescent behaviors are studied. In addition, guidelines for harmonic selection for specific layer separation is provided. The scattering matrices of different layers including proper harmonics are cascaded to obtain the GSM of the entire structure. The algorithm is verified through numerical examples including frequency selective surface (FSS) with different periodicities. Results showed good agreement with the results obtained from the FDTD simulation of the entire structure, while the new procedure provides significant saving in the computational time and storage memory.

Index Terms — FDTD, PBC, GSM, Floquet Harmonics, and FSS.

I. INTRODUCTION

Periodic electromagnetic structures are of great importance these days due to their applications in the design of frequency selective surfaces (FSS), electromagnetic band gap (EBG), corrugated surface, etc. Many periodic structures are built up of several layers [1]. Two approaches can be employed to analyze multilayered structures, one is to formulate and analyze a specific composite structure in its entirety [2]. This approach has serious practical limitations since the required amount of computation increases rapidly as the number of layers increases, and also since a complete new analysis is required every time a change is made in any layer. The other alternative is to compute the generalized scattering matrix (GSM) [3]-[6] for each layer and then obtain the total GSM of the structure by simple matrix calculations. This approach is more flexible and applicable to practical problems where several layers may be cascaded in arbitrary sequence. The cascade technique allows one to use different methods in computing the GSM for each layer of a multilayered structure. To the best of the authors’ knowledge, in most of the previous work the method of moments (MoM) and the finite element method (FEM) were used to compute the scattering parameters of each layer. In this work the finite-difference time-domain (FDTD) with constant horizontal wavenumber periodic boundary condition (PBC) [7]-[11] is used to compute the scattering parameters of each layer. The constant horizontal wavenumber FDTD/PBC approach offers many advantages, such as implementation simplicity, and the wide-band capability.

Usually, the GSM consists of scattering parameters of incident waves and their space harmonics, known as Floquet harmonics [12]-[13]. In multilayered periodic structures, the Floquet harmonics are particularly important due to the interactions between layers. A complete Floquet harmonic analysis is presented in this paper, where propagation and evanescent behaviors of harmonics are studied using FDTD method. In addition, guidelines are provided to select proper higher order harmonics for certain separation size. It is worthwhile to point out that the FDTD algorithm used in this paper is efficient for the harmonic analysis since the periodic boundary condition is handled by the constant horizontal wavenumber approach.

II. HYBRID FDTD/GSM METHOD

The GSM technique can take into account propagating and non-propagating modes and interactions between them (including cross-polarization effects). It describes the reflection and transmission properties of each layer by a scattering matrix for that layer, and uses a cascading process to obtain a scattering matrix for the overall structure. Each element in the scattering matrix is either a reflection or a transmission coefficient, which denotes the relationship between a scattered harmonic and one of the incident harmonics which excites it [14]-[16]. In principle, any desired level of solution accuracy can be obtained by using a sufficiently large matrix for each layer. In practice, the objective is to choose the matrix size large enough for good accuracy but small enough to keep the expenditure of computing resources within acceptable limits. The proposed algorithm can be summarized as follows:

1- Using the constant horizontal wavenumber FDTD/PBC, the scattering parameters of the first layer are calculated and the scattering matrix is constructed.

2- The scattering matrix of the first layer is transformed to a transmission matrix.

3- Step 1 and 2 are repeated for all the layers.
4- The total transmission matrix is calculated using matrix multiplication for all the transmission matrices.
5- The total transmission matrix is transformed to a scattering matrix, and all the scattering parameters are extracted from it.

III. FDTD/PBC FLOQUET HARMONIC ANALYSIS OF PERIODIC STRUCTURES

The presence of periodicity in the scatterer can lead to the appearance of far-field transmission and reflection at additional angles, often referred to as Floquet harmonics [17]. In this paper the periodicity is in x- and y-directions, and the generated harmonics will have wavenumbers as follows:

\[ k_x^{m,n} = k_x^i + \frac{2\pi m}{P_x}, k_y^{m,n} = k_y^i + \frac{2\pi n}{P_y}, \]

where \( m \) and \( n \) are the harmonic indices in x- and y-directions, respectively. In this analysis the harmonics are named using the following convention:

\[ M_{m,n} \ m = 0, \pm 1, \pm 2 \cdots, n = 0, \pm 1, \pm 2 \cdots \]  

These harmonics have cut-off frequencies, after which the harmonics start to propagate and it is no longer an evanescent harmonic. At \( k^2 = (k_x^{m,n})^2 + (k_y^{m,n})^2 \), the cut-off frequency occurs. The electric field of any mode can be in general written as (assume the y-component):

\[ \tilde{E} = E_{n,o} e^{-j(k_x^{m,n}x + k_y^{m,n}y + k_z^{m,n}z)} \hat{a}_y, \]

The magnitude of the harmonics (\( E_{m,n} \)) in (3) will be of certain value depending on the angle of incidence and the geometry of the periodic structure. To calculate the actual magnitude of different harmonics, the expression for the magnitude of the harmonic related to the total field can be stated as follows:

\[ E^{m,n}(\omega) = \frac{1}{P_x P_y} \int_0^{P_x} \int_0^{P_y} E(\omega, x, y) e^{j k_x^{m,n}x} e^{j k_y^{m,n}y} \ dx \ dy, \]

where \( k_x^{m,n}, k_y^{m,n} \) are given by (1), \( E(\omega, x, y) \) is the total frequency-domain field and \( x, y \) are the position of this electric field. Equation (4) can be re-written in a discrete form as:

\[ E^{m,n}(\omega) = \frac{1}{N_x N_y} \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} E(\omega, u \Delta x, v \Delta y) e^{j k_x^{m,n}(u \Delta x)} e^{j k_y^{m,n}(v \Delta y)}, \]

where \( N_x \) and \( N_y \) are the total number of cells in x- and y-directions, respectively, \( \Delta x \) and \( \Delta y \) are the cell size in x- and y-directions, respectively. The electric field \( E(\omega, u \Delta x, v \Delta y) \) is calculated using discrete Fourier transform (DFT), to transform the time-domain electric field at each cell into frequency-domain. This process requires saving all the time-domain components of electric fields at each cell. For instance, if the above calculation is done on a surface consisting of 30 × 30 cells over 2,500 time steps, for every time step at least two matrices (\( E_x, E_y \)) of the size 30 × 30 have to be stored. These matrices are then transformed to frequency-domain and the magnitude of different harmonics can be calculated using (5), which requires huge memory usage.

However, when the constant horizontal wavenumber approach is used, then \( k_x^{m,n} \) and \( k_y^{m,n} \) are constant and (5) can be directly transformed to time-domain as:

\[ E^{m,n}(t) = \frac{1}{N_x N_y} \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} \sum_{\nu=0}^{N_x \nu} \sum_{\mu=0}^{N_y \mu} E(t, u \Delta x, v \Delta y) e^{j k_x^{m,n}(u \Delta x)} e^{j k_y^{m,n}(v \Delta y)} \]

Using (6) the time-domain magnitude of each harmonic can be easily calculated in each simulation step. Thus, there is no need to store the field values at the 30 × 30 cells over the 2,500 time steps. Then these time-domain data of the harmonics are transformed to the frequency-domain using DFT. This feature of the constant horizontal wavenumber FDTD/PBC approach is considered as an important advantage due to the reduction in the memory usage.

FDTD Gap Determination Procedure:

1- Specify the periodicity, and the geometry of each layer: The periodicity and geometry of the layer is important to determine the cut-off frequencies and magnitudes of different harmonics.
2- Specify the frequency range of interest. The frequency range of interest is important to determine whether the harmonics are propagating or evanescent harmonics in this frequency range.
3- Specify the incident wave parameters (\( k_x^i \) and \( k_y^i \)). Use \( k_x^i \) and \( k_y^i \) to determine the cut-off frequencies of different harmonics. Any propagating harmonics in the frequency range of interest, should be considered whatever the gap size is.
4- Use (6) to determine the magnitudes of evanescent harmonics: Calculate \( k_z \) and use it together with the harmonic magnitude to study the decaying behavior of the evanescent harmonic with distance.
5- The gap size for neglecting the harmonic effect is calculated as the distance after which all evanescent harmonics magnitudes decays below -40dB compared to excitation electric field magnitude. The -40 dB threshold was concluded from different test cases for error less than 2%. Any other accuracy level can be achieved by changing the threshold value.

IV. NUMERICAL RESULTS

In this section, numerical examples to prove the validity of the hybrid FDTD/GSM approach are provided. The algorithm is used to simulate different test cases with a dipole and a square patch FSS with different periodicities. In all the test cases, the results of the cascaded technique are compared with the FDTD simulation of the entire structure [18]. The FDTD code was developed using MATLAB [19] and executed using a computer with an Intel Core 2 CPU 6700, 2.66 GHz with 2 GB RAM. As shown in Fig. 1, in this test case the multilayer geometry consists of two different FSS layers.
The first FSS structure consists of square patch elements, with side of length 6 mm. The substrate has thickness of 6 mm and relative permittivity \( \varepsilon_r = 2.2 \). The second FSS structure consists of dipole elements of length 12 mm and width 3 mm. The structure is illuminated by a normal incident plane wave \( (k_x = k_y = 0 \text{ m}^{-1}) \), the frequency range of interest is 0-16 GHz. The FDTD grid cell size is \( \Delta x = \Delta y = \Delta z = 0.5 \text{ mm} \).

It is important to first determine the distance \( d \) after which all the harmonics reach -40 dB from the magnitude of the incident electric field. Using the gap determination procedure described in Section III this distance can be easily determined. A gap of 13.11 mm was found to be enough to neglect the higher harmonics effects. To validate the cascading technique and the gap determination procedure the structure is analyzed using the cascading technique (in the cascading technique only one unit cell from each layer is used) and compared with the FDTD simulation of the entire structure as shown in Fig. 2. The maximum relative error in the case of \( d = 15 \text{ mm} \) is less than 0.3%. The relative error is calculated as follows:

\[
\text{error}(f) = \frac{\| r_{\text{entire}}(f) \| - \| r_{\text{cascaded}}(f) \|}{\max(\| r_{\text{entire}}(f) \|)} \times 100\%.
\]  

(7)

The computational time using the cascaded technique is less than the computational time for the entire structure especially with large gaps. In addition, to simulate the entire structure many unit cells are needed for each layer. However, by using the cascading technique, only one unit cell is simulated for each layer which reduces the computational time dramatically. The computational time for the cascaded case is 8 minutes (for calculating S-parameters of the FSS layers and the air gap and calculating the total GSM), while for the simulation of the entire structure it takes 130 minutes due to the large domain simulated. Moreover, for the cascading case the domain size for the cascading case is equal to 43,200 cells \((30 \times 30 \times 48)\), while for the entire structure the domain size is 280,800 cells \((60 \times 60 \times 78)\), which illustrate the efficiency of the hybrid FDTD/GSM algorithm with respect to the memory usage. In addition, the entire structure simulation requires large number of time steps to generate stable results.

To study the same structure but with a small gap, the algorithm is used to analyze the structure with a gap size equal to 3.5 mm. It was noticed that the higher harmonics transmitted from the first layer will decay below -40 dB at a distance of 2.5 mm, so for the gap size of 3.5 mm only two harmonics need to be considered in the analysis to get accurate results from the cascading technique (\( M_{1,0}, M_{-1,0} \) of the second layer). As long as three modes are included in the analysis, the S-matrix of each layer will be of the size \( 6 \times 6 \). Figure 3 shows the results of the co-polarized reflection coefficient using cascading technique with only dominant mode, and with dominant mode plus the harmonics \( (M_{1,0}) \) and \( (M_{-1,0}) \) of the second layer. The results are compared with the FDTD simulation of entire structure. It could be noticed from Fig. 3 that when the effect of the \( (M_{1,0}) \) and \( (M_{-1,0}) \) harmonics are taken into consideration accurate results are obtained. The maximum relative error in the case of cascaded technique with
only the dominant mode is 3%, while in the case of the harmonics (M_{1,0}) and (M_{-1,0}) included a maximum relative error of 0.3% is obtained.

V. CONCLUSION

In this paper, an efficient hybrid FDTD/GSM technique is described. In this technique constant horizontal wavenumber FDTD/PBC approach is used to compute the scattering parameters of each layer, after which the scattering matrix of the entire structure is calculated using the cascading technique. In addition, two procedures were described: one is to study the behavior of different harmonics (evanescent and propagating) using constant horizontal wavenumber FDTD/PBC approach, which dramatically reduces the memory usage; the other procedure is used to determine the proper gap size for considering the higher harmonics effects. The validity of the algorithm was verified through two numerical examples including frequency selective surfaces (FSS) with different periodicities. The numerical results of the developed approach show good agreement with the results obtained from the direct FDTD simulation of the entire structure, while the proposed algorithm saves computational time and memory usage.

REFERENCES