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DIFFRACTION BY A DIELECTRIC-LOADED
DOUBLE WEDGE WITH DENTED EDGES

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SUMMARY

This paper considers the problem of diffracted electromagnetic waves by a loaded double conducting wedge with dented edges depicted in Figure 1. The loading is of the form of a dielectric cylinder whose circumference coincides with the circular arcs of the dents. The axis of the cylinder is therefore the line of intersection of the two upper wedges. The dielectric medium is assumed linear, homogeneous, isotropic, and free from losses, and it therefore characterized by the real scalars permittivity ε, permeability μ, and wave number k. The medium of the upper and lower sectors are free spaces characterized by the constitutive parameters εo, μo, and k. The analysis utilizes a field equivalence theorem by which the fields in the different regions are determined in terms of equivalent magnetic current placed on the dielectric interface while they are covered by perfect electric conductors. The basic formulation and the specific of the TE case are given below. The TM case and some of the results obtained for both cases will be discussed at the meeting.

BASIC FORMULATION

Let the excitation of the dielectric-loaded double dented wedge be a line source situated in the upper sector, \( \sigma > s > \sigma + \delta > \delta > 0 \), at \( \rho = \rho_0 \) and \( \phi = \phi_0 \). The case of plane wave excitation can be treated as a special case by letting the line source extends to infinity. The total field, incident plus scattered, must have zero electric field components tangent to the conducting wedges, and continuous tangential electric and magnetic fields across the dielectric interfaces. A field equivalence theorem is used to divide the problem into three problems for the upper and lower sectors and the dielectric cylinder as follows. Let the exciting field be the field produced by the line current while the dielectric interfaces are covered by perfect conductors. This field, often referred to as the short-circuit fields, is denoted \( \mathbf{E}_0 \) and \( \mathbf{H}_0 \). Furthermore, let \( \mathbf{E}_o(H_0, \mathbf{H}_o) \) and \( \mathbf{H}_o(H_0, \mathbf{E}_o) \) be, respectively, the fields produced in the upper sector and in the dielectric cylinder by the magnetic current

\[
\mathbf{J}_s = z_0 \mathbf{E}
\]  

on the dielectric interface \( \sigma > s > \sigma + \delta > \delta > 0 \), and let \( \mathbf{H}_o(H_0, \mathbf{E}_o) \) and \( \mathbf{E}_o(H_0, \mathbf{H}_o) \) be, respectively, the fields produced in the lower sector and in the dielectric cylinder by the magnetic current

\[
\mathbf{J}_s = z_0 \mathbf{H}
\]  

on the dielectric interface.
on the dielectric interface $\rho = \infty$, $\sigma - \alpha \geq \frac{1}{2} r + a$, while they are covered by perfect conductors. In (1) and (2), $E$ is the total electric field on the corresponding interface. By the field equivalence theorem [1, Section 3-1], the total field in upper sector is identical with $(\nabla \times \mathbf{E}^\infty (\mathbf{H}_1) + \mathbf{E}^\infty \times \mathbf{H}^\infty (\mathbf{H}_1))$, the field in the dielectric cylinder is identical with $(-\nabla \times \mathbf{E}_1 + \mathbf{H}_1)$, $-\nabla \times \mathbf{E}_2 + \mathbf{H}_2)$, and the field in the lower sector is identical with $(\nabla \times \mathbf{E}_2, \mathbf{H}^\infty (\mathbf{H}_2))$. The tangential components of the total electric field do vanish tangent to the conducting wedge by construction. Furthermore, they are continuous across the dielectric interfaces by virtue of placing magnetic current sheets of equal amplitudes and opposite signs on the opposite sides of the covered interfaces. The continuity of the tangential components of the magnetic field across the interfaces requires that

\[
\mathbf{a}_e \times \left[ \mathbf{E}^\infty (\mathbf{H}_1) + \mathbf{E}^\infty \times \mathbf{H}^\infty (\mathbf{H}_1) \right] = -\mathbf{a}_h \times \mathbf{H}^\infty, \quad \rho = \infty, \quad -\alpha \leq \phi \leq \alpha
\]

(3)

\[
\mathbf{a}_e \times \left[ \mathbf{E}^\infty (\mathbf{H}_2) + \mathbf{E}^\infty \times \mathbf{H}^\infty (\mathbf{H}_2) \right] = 0, \quad \rho = \infty, \quad 0 \leq \phi \leq \alpha
\]

(4)

which is a coupled pair of equations for the equivalent magnetic currents $\mathbf{H}_1$ and $\mathbf{H}_2$.

**THE TE CASE**

The sources in the TE case is a magnetic line current of unit strength. The field produced by such a current source can have only a $z$-component of magnetic field and no such component of electric field. It then follows from (1) and (2) that the equivalent magnetic currents $\mathbf{H}_1$ and $\mathbf{H}_2$ have only a $z$-component that does not vary with $z$; viz.,

\[
\mathbf{H}_{1,2} = \mathbf{H}_{1,2} (\phi) \mathbf{a}_z
\]

(5)

where primed coordinates are used to locate source points. Thus, all the sources involved in the equivalent problems are $z$-directed uniform magnetic line currents. The magnetic field in any of the equivalent problems can then be derived from a $z$-directed electric vector potential $\mathbf{A}$ [1, Section 5-1] as

\[
\mathbf{H}_z = -\nabla \times \mathbf{A}_z (\mathbf{A}, \phi)
\]

(6)

The $z$-component of the electric vector potential $\mathbf{A}_z$ can itself be determined in terms of the Green's functions of the magnetic type $\mathbf{G}^{\text{MV}}$ of a doubly infinite uniform magnetic line current. The magnetic field in any of the equivalent problems can then be derived from a $z$-directed electric vector potential $\mathbf{A}_z$ as

\[
\mathbf{E}_z (\mathbf{A}, \phi) = \mathbf{G}^{z} (\mathbf{A}, \phi) \mathbf{A}_z
\]

(7)

\[
\mathbf{E}_z (\mathbf{A}, \phi) = \int_{0}^{a} \mathbf{G}_z (\phi, \phi') \mathbf{A}_z (\mathbf{A}, \phi') \, d\phi'
\]

(8)
\[ D^p(z, \theta) = \int_{\gamma} N^{B^p}(z, \theta', s') \, ds' \]

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\[ \Omega^p(z, \theta', \phi') = \int_{0}^{2\pi} N^{B^p}(z, \theta, \phi') \, d\phi' \]

In (9) and (10)

\[ \Omega^p(z, \theta', \phi') = \]

\[ \ \frac{1}{2} \sum_{n=0}^{N} \left[ \sigma^{(2)}(n) \frac{J_n(z)}{J_n(-z)} - J_n(-z) \right] \cos^{2n} \theta' \cos^{2n+1} \phi' \]

\[ \Omega^p(z, \theta', \phi') = \]

\[ \ \frac{1}{2} \sum_{n=0}^{N} \left[ \sigma^{(2)}(n) \frac{J_n(z)}{J_n(-z)} - J_n(-z) \right] \cos^{2n} \theta' \cos^{2n+1} \phi' \]

where \( \sigma^{(2)} \) and \( J \) are, respectively, the Hankel function of the second kind and Bessel function of the first kind, primes denote derivatives of the functions with respect to their arguments, and \( \nu_n \) is the Neumann's number (\( \nu_0 = 1 \) and \( \nu_n = 2 \) for all \( n \geq 1 \)). Furthermore, the order of the Hankel and Bessel functions in (11) is \( \nu(z) \), where \( \nu(0) = \pi / (z - 2\pi) \), and the signs and plus signs correspond, respectively, to the upper and lower sectors. Substituting (8), (9) and (10) into the coupled pair (3) and (4), there then result

\[ D^p(z, \theta) = \int_{\gamma} N^{B^p}(z, \theta', s') \, ds' \]

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\[ \Omega^p(z, \theta', \phi') = \int_{0}^{2\pi} N^{B^p}(z, \theta, \phi') \, d\phi' \]

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\[ \sum_{\alpha} \int_{0}^{\infty} \left[ R_{2}(\phi') \left( R_{2}(\phi | s, \phi') + \varepsilon_{e} \int_{-\infty}^{\infty} \frac{d\varepsilon'}{\varepsilon'} \right) \right] d\phi' = 0, \]

\[ \alpha = 1 \rightarrow \infty \]  \hspace{1cm} (14)

THE GALERKIN'S SOLUTION

Solution of the coupled pair of equations (13) and (14) in the TE case can readily be carried out using the Galerkin's method [2, Section 1.2]. This is accomplished by expanding the unknown currents \( M_1 \) and \( M_2 \) in terms of complete sets of orthogonal functions defined on their respective domains, then testing the resulting equations with the same functions. The details of the solution will be presented at the meeting.

REFERENCES


Figure 1. A dielectric-loaded double wedge with dented edges.