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ILL Number: 90767915

OCLC #: 22416807

Journal Title: Proceedings ; energy and information technologies in the Southeast ; Southeastcon, April 1-4, 1990, conference and exhibit, Doubletree Hotel, New Orle
Volume: 3
Issue:
Month/Year: 1990
Pages: 996-1001

Article Author:
Article Title: Elsherbeni, A.Z.; Electromagnetic scattering from a circular cylinder of homogeneous dielectric coated by a dielectric shell with a permittivity profi

YES  NO  Reason:
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Electromagnetic Scattering From A Circular Cylinder Of Homogeneous Dielectric Coated By A Dielectric Shell With a Permittivity Profile In The Radial And Azimuthal Directions — Even TM Case

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ABSTRACT

Equations are formulated and numerical results are presented for electromagnetic scattering from an infinitely long circular cylinder of homogeneous dielectric coated with a dielectric shell. The permittivity of the dielectric shell varies as \( r^2 \) in the radial direction and as a weighted cosine in \( \phi \). Plane waves with an electric field parallel to the cylinder axis constitute the incident field, restricting the analysis to the transverse magnetic (TM) case.

Fields inside the homogeneous cylinder are represented by an infinite summation of Bessel functions with unknown coefficients, and scattered fields external to the shell are represented by an infinite summation of Hankel functions with unknown coefficients. The choice of dielectric profile for the shell allows the field inside the shell to be represented as two infinite sums of Mathieu functions with unknown coefficients. Enforcing boundary conditions at the two interfaces, using sinusoidal expansions for the Mathieu functions, and employing the orthogonality of the sinusoidal functions and the orthogonality of the Mathieu functions results in a matrix equation independent of \( \phi \) which can be evaluated to find the coefficients that define the scattered field. The method is computationally efficient with relatively small matrices required for convergence.

There are four independent parameters associated with the dielectric shell: the thickness of the shell; \( \epsilon_\phi \), which is proportional to the relative permittivity; the relative weight of the azimuthal variation; and the frequency of the azimuthal variation. Investigation reveals that a judicious choice of parameters allows the scattered field to be modified to:
1) Minimize the back-scattering cross section, or
2) Minimize the forward-scattering cross section, or
3) Minimize both the forward- and back-scattered cross section, or
4) Otherwise manipulate the scattered field.

Potential applications include dielectric target supports for field measurements, beam splitters and electromagnetic lenses. Analysis for TE incidence and realization of the dielectric profile remain to be investigated.

INTRODUCTION

Dielectric materials can be used to modify the radiation characteristics of radiators and the radiation fields of scatterers. A number of theoretical and experimental investigations have been carried out to study control of antenna characteristics by dielectrically loading conventional antennas and reflector surfaces [1-5]. However, previous investigations have mostly been restricted to dielectric materials with constant permittivity across the whole dielectric or permittivity that varies in only one dimension. Since radiation characteristics were improved in many cases due to the addition of dielectric material, investigations which extend current knowledge are potentially of interest to those engaged in the design and application of antennas. There is no formal solution yielding time-harmonic electromagnetic fields in regions where the permittivity, \( \epsilon \), is a general function of position; however, specific forms of inhomogeneity may yield rigorous solutions for the electromagnetic field [6-8]. This paper treats one such case where \( \epsilon \) is inversely proportional to \( r^2 \) and is a sinusoidal function of \( \phi \) in the circular cylindrical coordinates.

In the present work, the problem of scattering of an incident TM plane wave by a circular dielectric cylinder coated with a dielectric shell of inhomogeneous material is considered. Field components in free space are expressed in terms of cylindrical harmonic functions with unknown scattering coefficients. The permittivity of the dielectric coating is assumed to be proportional to \( r^2 \) and is a function of \( \cos m \phi \) where \( m \) is an integer. This permittivity profile allows the unknown electric field in the dielectric region to be represented by a series of cosine-elliptic Mathieu functions of even integer orders with unknown expansion coefficients. The unknown expansion coefficients are determined by enforcing boundary conditions and using the orthogonal properties of the trigonometric functions and of the Mathieu functions. The far scattered field is then evaluated and numerical results for the scattered field versus different geometrical parameters are obtained and presented.

BASIC FORMULATION

Consider a homogeneous dielectric cylinder of radius \( \rho_a \), loaded by a dielectric shell with outer and inner radii \( \rho_a + b \) and \( \rho_a \), respectively, and a permittivity variation of the form

\[
\epsilon_\phi(\rho, \phi) = \epsilon_\phi \left( \frac{\rho_a}{\rho} \right)^2 (1 - \delta \cos m \phi) = \epsilon_\phi \epsilon_{\phi} \left( \rho, \phi \right)
\]

in which \( \epsilon_\phi \) and \( \delta \) are constants, \( m \) is an integer, \( \epsilon_\phi \) is the permittivity of free space and the subscript \( \phi \) denotes the inhomogeneous dielectric region. Thus, the dielectric medium is inhomogeneous in two coordinate variables, \( \rho \) and \( \phi \) in the circular cylindrical coordinates.

The configuration is completely symmetric around \( \phi = \phi_0 \). If \( \phi = \phi_0 \), \( n = 0, 1, 2, \ldots, 2m \), and this paper restricts the angle of incidence so that only even cases are studied. The problem is two dimensional, since the field components are independent of \( \z \), and the time dependence \( \exp(jut) \) is assumed and suppressed.
where the superscript d denotes fields in the inhomogeneous dielectric region. By means of separation of variables based on the assumption that

\[ E_z(\rho, \phi) = \rho^{-1/2} \phi(\phi), \]  

where \( \alpha \) is the separation constant, equation (4) becomes

\[ \frac{d^2 \phi}{d\phi^2} + \left[ a^2 + \left( k_0 a \right)^2 \epsilon_a (1 - \delta \cos \phi) \right] \phi = 0 \]  

(5)

Letting \( \Phi = 2\phi \), equation (5) reduces to

\[ \frac{d^2 \Phi}{d\Phi^2} + \left( a - 2q \cos 2\Phi \right) \Phi = 0 \]  

(6)

where

\[ a = \frac{k_0}{\lambda} \left[ a^2 + \left( k_0 a \right)^2 \epsilon_a \right], \]  

and \( q = \frac{2}{m} \left( k_0 a \right)^2 \epsilon_a \delta \).  

(7)

Equation (6) represents a Mathieu differential equation and since the problem is symmetric with respect to the \( \phi = 0 \) plane and \( E_z^d \) has period \( 2\pi \) in \( \phi \), only even solutions of \( \Phi(\phi) \) should be considered. Hence, a superposition of all possible solutions satisfying these conditions will be

\[ E_z^d(\rho, \phi) = E_0^d \sum_{n=0}^{\infty} \left[ d_{2n}^0 \rho^{2n} + b_{2n}^0 \rho^{-(2n+1)} \right] c_{2n}(\phi, q) \]

\[ + \left( \frac{e_n}{2n+1} \rho^{2n+1} + b_n^e \rho^{-(2n+1)} \right) c_{2n+1}(\phi, q) \]  

(8)

where the characteristic number, \( a_n \), of the Mathieu function \( c_{2n}(\phi, q) \) is given by

\[ a_n = \frac{\frac{m}{2} - a - \left( k_0 a \right)^2 \epsilon_a}{a} \]  

(9)

and \( d_{2n}^0 \) and \( b_{2n}^0 \) are unknown expansion coefficients. The cosine elliptic functions \( c_{2n}(\phi, q) \) and \( c_{2n+1}(\phi, q) \) have periods of \( \pi \) and \( 2\pi \), respectively [10]. The magnetic field component \( H_\phi^d \) in the inhomogeneous dielectric region is then given by

\[ H_\phi^d(\rho, \phi) = \frac{E_0^d}{j \omega \mu_0} \sum_{n=0}^{\infty} \left[ a_n \left( e_n^d \rho^{2n+1} - b_n^e \rho^{-(2n+1)} \right) c_{2n+1}(\phi, q) \right. \]

\[ + \left. b_n^e \rho^{2n+1} c_{2n+1}(\phi, q) \right] \]  

(10)

Since the field should be finite at \( \rho = 0 \), the field in the interior region can be expressed as an infinite series of Bessel functions of the first kind, \( J_n(k_0 \rho) \). Thus,

\[ E_z(\rho, \phi) = E_0 \sum_{n=0}^{\infty} \epsilon_n \alpha_n J_n(k_0 \rho) \cos \phi \]  

(11)

and as a result

\[ H_\phi(\rho, \phi) = \frac{E_0}{j \omega \mu_0} \sum_{n=0}^{\infty} \epsilon_n \alpha_n J'_n(k_0 \rho) \cos \phi \]  

(12)
where $k_e$ is given by $k_e = \frac{\mu_0}{\varepsilon_c}$, where $\varepsilon_c$ is the permittivity of the core.

**SOLUTION OF THE EXPANSION COEFFICIENTS**

The unknown expansion coefficients $a_n$, $b_n$, and $c_n$ can be evaluated from the continuity of the tangential components of the electric and magnetic fields at $r = \rho_a$ and $r = \rho_b$. Setting $E_z$ equal to $E_x$ at $r = \rho_a$ yields

$$
\sum_{n=0}^{\infty} e_n \left( J^{(2)}_n(k \rho_a) + c_n H_n^{(2)}(k \rho_a) \right) \cos n\phi = 0
$$

$$
= \sum_{n=0}^{\infty} (a_n^{2n} e^{-\alpha_2 n} + b_n^{2n} e^{\alpha_2 n} - a_n^{2n-1} e^{-\alpha_2 n+1} - b_n^{2n+1} e^{\alpha_2 n+1}) \cos (n+1) \phi.
$$

The right hand side of (13) is then multiplied by $ce^2_t(\psi, q)$, and the left hand side is multiplied by the expansion of $ce^2_t(\psi, q)$, i.e.

$$
ce^2_t(\psi, q) = \sum_{r=0}^{\infty} A_r^{2\psi} \cos 2\phi r,
$$

where the coefficients $A_r^{2\psi}$ can be computed once $q$ and the characteristic number $a^{2\psi}$ are known [11].

Integrating both sides of the resulting equation from 0 to $2\pi$ and using the orthogonal properties of the Mathieu functions and of the cosine functions [12] gives

$$
a^{2\psi} e^{-\alpha_2 n} + b^{2n} e^{\alpha_2 n} = \sum_{r=0}^{\infty} \left( J_{2r+1}(k \rho_a) + c_{2r+1} H_{2r+1}^{(2)}(k \rho_a) \right) \cos 2\phi r
$$

$$
= \sum_{r=0}^{\infty} A_r^{2\psi} \cos 2\phi r.
$$

Again multiplying (13) by $ce^2_{t+1}(\psi, q)$ and using the expansion

$$
ce^2_{t+1}(\psi, q) = \sum_{r=0}^{\infty} A_{r+1}^{2\psi} \cos (2\phi r + 1)\phi
$$

and integrating from $0$ to $2\pi$ gives

$$
da^{2\psi+1} e^{-\alpha_2 n} + b^{2n+1} e^{\alpha_2 n} = \sum_{r=0}^{\infty} \left( J_{2r+1}(k \rho_a) + c_{2r} H_{2r}^{(2)}(k \rho_a) \right) \cos 2\phi r
$$

$$
+ c_{2r} H_{2r+1}(k \rho_a) A_r^{2\psi+1}
$$

where $x = 0, 1, 2, \ldots$ and $r = 0, 1, 2, \ldots$

Equations (15) and (17) are combined to form one matrix equation, i.e.

$$
[U][D] + [W][B] = [S] + [Z][C]
$$

where each element of the column matrices [D], [B], [C] is one of the unknown expansion coefficients $a_p$, $b_p$, and $c_p$, respectively. The matrices [U] and [W] are square and diagonal, and each of their diagonal elements is given, respectively, by

$$
u_{1,1} = \rho_a^2 a_p a_p^{-1} A_p
$$

$$
u_{1,1} = \rho_a^2 a_p a_p^{-1} A_p
$$

$$
u_{1,1} = \rho_a^2 a_p a_p^{-1} A_p
$$

Each element of the column matrix $[S]$ is denoted by $s_{1}$ and is given by

$$
s_{1} = \sum_{i=0}^{\infty} e_{1}^{i} j_{i}(k \rho_a) A_i^{f}
$$

while [2] is a square matrix with elements given by

$$
z_{t,p} = e_{p}^{(2)} j_{p}(k \rho_a) A_p^{f},
$$

and the integers $p$ have the values 0, 1, 2, ... .

Setting $H_\rho$ equal to $H_\rho^d$ at $\rho = \rho_b$ and following a similar procedure yields

$$
[C] = [F] + [T][D] + [X][B]
$$

where [F] is a column matrix in which each element $f_p$ is given by

$$
t_p = \frac{-1}{T_{p}} J_{p}(k \rho_a) H_{p}^{(2)}(k \rho_a)
$$

and the elements of the square matrices [T] and [X] are given, respectively, by

$$
t_{p,q} = \rho_b^2 e_{p} e_{q} A_{p}^{f} A_{q}^{f},
$$

Applying the boundary conditions that require continuity of $E_z$ and $H_\rho$ at $\rho = \rho_b$, we obtain after some mathematical manipulations,

$$
[U][2] + [Y][2] = [D] = [Y][X] [X] - [W] [B]
$$

where [U] and [W] are diagonal matrices and each of their elements is denoted by $u_{1,1}$ and $u_{2,2}$, respectively and are given by

$$
u_{1,1} = \rho_b^2 a_p a_p^{-1} A_p
$$

The elements of the square matrices [Y], [T2] and [X2] are denoted by $y_{p,q}^{f}$, $c_{p}^{2}$, and $x_{p,q}^{2}$, respectively and are given by

$$
y_{p,q}^{f} = \frac{-1}{T_{p}} J_{p}(k \rho_b) H_{p}^{(2)}(k \rho_b)
$$

$$
t_{p,q}^{2} = \rho_b^2 e_{p} e_{q} A_{p}^{f},
$$

Furthermore, the coefficients $a_p$ are given by

$$
a_{p} = \frac{-1}{T_{p}} J_{p}(k \rho_b) H_{p}^{(2)}(k \rho_b)
$$

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$$
a_{p} = \frac{-1}{T_{p}} J_{p}(k \rho_b) H_{p}^{(2)}(k \rho_b)
\begin{equation}
\rho = \frac{1}{p_k} \sum_{n=0}^{\infty} \alpha_n \left( d_n e^a n - \alpha_n \right) \rho_b n^{a-1} n^{-1} \rho_p n^a n^{-1} \quad (31)
\end{equation}

Equations (18), (22) and (26) are then rearranged to find the unknown expansion coefficients \( b_n^e \) and \( b_n^o \). The results are

\begin{equation}
[D] = \begin{bmatrix} X_3 & X_4 & X_5 & X_6 \end{bmatrix}^{-1} \begin{bmatrix} S + Z \end{bmatrix} \begin{bmatrix} F \end{bmatrix},
\end{equation}

and \([B] = \begin{bmatrix} X_5 & X_6 \end{bmatrix} [D],\)

where \([X_3] = [U] - [Z] [T],\)

\([X_4] = [Z] [X] - [W],\)

\([X_5] = \begin{bmatrix} Y_1 & Y_2 \end{bmatrix}^{-1}\)

and \([X_6] = [U_2] - [Y] [T_2].\)

Once the unknowns \( d_n^e \) and \( b_n^o \) are computed, we can obtain the unknowns \( u_n \) and \( a_n \) from equations (22) and (31), respectively.

**RESULTS**

This section presents results from the procedure described above. Scattered fields are compared to solutions obtained using the method of moments and to exact solutions in limiting cases. Finally, results are presented to illustrate the effect of loading dielectric cylinders with a shell of this dielectric profile. In all the following figures, the matrix order, \( N \), of this solution is determined when \(|A_{2N}| / \max |A_{2r}| \leq 10^{-12}\). Convergence tests for a wide range of parameters indicate that the solution has converged at this matrix size.

All the figures show the scattered field from a plane wave incident at \( \theta_0 = 0^\circ \). In Figure 2, \( \delta \) is set to .001 and the shell thickness is .001 \( \lambda \) to minimize the effect of permittivity variations in the shell, and results are compared to an exact solution, with good agreement. Figures 3 and 4 present results from the shell only with an air core. In Figure 3, the azimuthal variation is again minimized and this solution is compared to one obtained with the method of moments. In Figure 4 a very thin (.005 \( \lambda \)) shell is used to minimize the effect of the radial variation, but the azimuthal variation is significant. For Figures 3 and 4 the moment method solution uses subshells of constant permittivity [13]. This method of modeling permittivity variations has been a source of considerable difficulty in obtaining good agreement between this solution and moment method solutions - the results shown are among the best obtained so far. The moment method solution in Figure 3 uses 4 radial shells, each with 50 azimuthal subshells; in Figure 2, 2 radial shells, each with 100 subshells, are used.

Figure 5 compares results from this solution of a core/shell to exact results for the core only. Two different dielectric shell profiles are presented; both minimize the back scattered cross section, while one results in small scattered fields in every direction. The latter geometry must result in significant energy disposition inside the cylinder/shell, but computation of internal fields remains an area for future research. In Figure 6 scattered fields from a core/shell are compared to an

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**Figure 2.** Comparison with exact solution for \( \rho_b = 1.0 \) and \( \rho_r = 1.01 \lambda \). Exact: \( \epsilon_r = 5, \epsilon_a = 2.02 \). Shell: \( \epsilon_r = 5, \epsilon_a = 2, \delta = .001, \) and \( m = 1. \)

**Figure 3.** Comparison with moment methods - no azimuthal variation. \( \rho_b = 0.25 \lambda, \rho_r = 0.27 \lambda, \epsilon_r = 1, \epsilon_a = 4, \delta = .001, m = 1. \) Moment method has 4 radial subshells with 50 points each.

**Figure 4.** Comparison with moment methods - very thin shell. \( \rho_b = 0.25 \lambda, \rho_r = 0.255 \lambda, \epsilon_r = 1, \epsilon_a = 4, \delta = 0.2, m = 1. \) Moment method has 2 radial subshells with 100 points each.
exact solution for the core only. In this case, the effect of the shell is to increase the back scattered cross section. Figure 7 compares two different shell profiles to an exact solution for the core only.

Here, the forward scattered cross section is increased, so that the addition of the shell has a focusing effect.

CONCLUSION

A solution to even TM scattering from a dielectric circular cylinder of constant permittivity coated with a class of inhomogeneous dielectric shells is presented. The shell permittivity varies continuously as $\rho^{-2}$ in the radial direction and as $\cos(\alpha z)$ in the azimuthal direction. Numerical results are obtained when the resulting infinite matrices are truncated and unknown coefficients are determined.

Addition of the dielectric shell can significantly alter scattered field when compared to an uncoated dielectric core.

REFERENCES


Mark Tew was born in Meridian, Mississippi, in 1949 and attended the University of Mississippi, receiving the BSCE degree in 1971 and an M.S. in Engineering Science in 1973. After working for Kaman Sciences Corp., Colorado Springs, Co., and TRW Systems Group, Redondo Beach, Ca., he attended the University of Illinois and received the PhD in 1979. Dr. Tew joined the faculty of the Department of Electrical Engineering at the University of Mississippi that year, where he is now an Associate Professor. Dr. Tew's interests include numerical solution techniques, antennas, microprocessors, volleyball and environmental activism.