AN EFFICIENT FINITE DIFFERENCE METHOD FOR FINDING THE ELECTRIC POTENTIAL IN REGIONS WITH SMALL PERTURBATIONS

Richard K. Gordon*, Mark D. Tew, and Atef Z. Elsherbeni
Department of Electrical Engineering
University of Mississippi
University, MS 38677

INTRODUCTION

Alternative derivative formulas [1] are used in the finite difference method with a nonuniform mesh to determine the difference field that arises as the result of the introduction of a small obstacle into a region in which an electric field is present. Unlike previous techniques employing integral equation formulations [2-3], this is a partial differential equation approach.

DIFFERENCE POTENTIAL TECHNIQUE

We consider a situation such as that shown in Figs. 1 and 2. The electric potential along the sides and top of a rectangular region is zero; along the bottom of the rectangle it satisfies the boundary condition, $V(x,0)=\sin(3\pi x/2)$. Before the addition of any obstacle, the original potential existing in the rectangular region can be determined analytically. After the introduction of an obstacle such as the small rectangular piece of perfect electric conductor (PEC) shown in Fig. 1 or the dielectric cross shown in Fig. 2, the perturbed potential is determined numerically. One way of doing this is to solve for the perturbed potential directly using the conventional finite difference method. Since the perturbed potential is varying significantly throughout the problem domain, this approach would require the use of a finite difference mesh that is rather dense everywhere in the rectangular region. In order to avoid this rather costly calculation, we consider a different approach. We instead solve for $V_{difference}$ defined as: $V_{difference} = V_{perturbed} - V_{original}$. Here, $V_{original}$ is the potential in the rectangular region before the introduction of the obstacle and $V_{perturbed}$ is the potential afterwards. Because this difference potential is insignificant away from the obstacle, it is possible to numerically determine $V_{difference}$ using a mesh that is dense near the obstacle but sparse away from it. That is, $V_{difference}$ can be accurately determined using a finite difference mesh having far fewer nodes than the mesh that must be used in the direct numerical determination of $V_{perturbed}$. Thus, the use of this method results in a significant reduction in the amount of computer time and memory required. Once $V_{difference}$ has been determined, $V_{perturbed}$ is found by adding $V_{original}$, which can be conveniently determined analytically, to $V_{difference}$.

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NUMERICAL RESULTS

In Fig. 1, the length and height of the PEC obstacle are 1/28 the length and height of the rectangular region. The perturbed problem was solved directly using a dense mesh having 1653 nodes. The numerical results obtained for the perturbed potential along the horizontal line abutting the bottom of the obstacle are plotted as the dotted line in Figs. 3(a) and 3(b). The difference technique described above was employed using a sparse mesh having 231 nodes. The numerical results obtained for the sum of the original and difference potentials along the horizontal line abutting the bottom of the obstacle are plotted as the solid line in Fig. 3(a). There is excellent agreement with the results obtained from the direct solution of the perturbed problem using the denser mesh. The results obtained from an attempt to directly solve the perturbed problem with the same sparse mesh used in the determination of the differential potential in the technique described above are plotted as the solid line in Fig. 3(b). Clearly, the mesh is too sparse to be used in the direct solution of the perturbed potential. Thus, the use of the method described above has permitted the determination of the perturbed potential with a sparser finite difference mesh than the one that must be used in a direct solution technique.

In Fig. 2, the center of the dielectric cross is at the center of the rectangular region. The lengths of the entire horizontal and vertical segments of the dielectric cross are the same and are equal to 1/7 the length of the rectangular region. The thickness of each segment is 1/28 the length of the rectangular region. The relative permittivity is 4.0. The perturbed problem was solved directly using a dense mesh having 1653 nodes. The numerical results obtained for the perturbed potential along the horizontal line through the center of the cross are plotted as the dotted line in Figs. 4(a) and 4(b). The technique described above was employed using a sparse mesh having 399 nodes. The numerical results obtained for the sum of the original and difference potentials along the horizontal line through the center of the cross are plotted as the solid line in Fig. 4(a). As in the case of the PEC obstacle, there is excellent agreement with the results obtained from the direct solution of the perturbed problem using the denser mesh. The results obtained from an attempt to directly solve the perturbed problem with the same sparse mesh used in the determination of the differential potential in the technique described above are plotted as the solid line in Fig. 4(b). Once again, although the mesh is adequate for the determination of the difference potential, it is too sparse to be used in the direct solution of the perturbed potential. So, again, the use of the method described above has permitted the determination of the perturbed potential with a sparser finite difference mesh than the one that must be used in a direct solution technique.

CONCLUSIONS

The electric potential in a region containing small perturbations can be efficiently determined by using a finite difference technique with a nonuniform mesh plus knowledge of the original potential in the unperturbed problem. This can be applied to PEC obstacles, dielectric obstacles, or combinations of both.

ACKNOWLEDGEMENT

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REFERENCES

[1] R. Gordon, J. F. Lee, and R. Mittra, "A second-order accurate method on a non-uniform mesh in the finite difference frequency domain approach to EM scattering problems," North American Radio Science Meeting (URSI), London, Ontario, 1991, p.240.

[2] Mark D. Tew and Leonard L. Tsai, "A method toward improved convergence of moment-method solutions," IEEE Proceedings, vol. 60, pp.1436-1437, November 1972.

[3] A. Z. Elsherbeni and M. Hamid, "Diffraction properties of a class of double wedges," Can. J. Phys., vol. 65, pp. 16-22, January 1987.

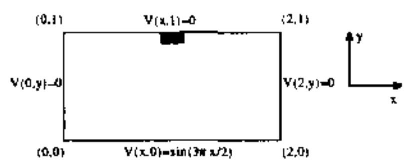


Figure 1. Geometry for PEC obstacle

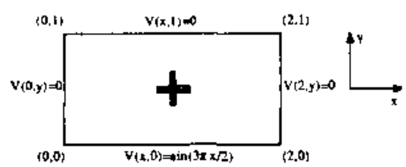
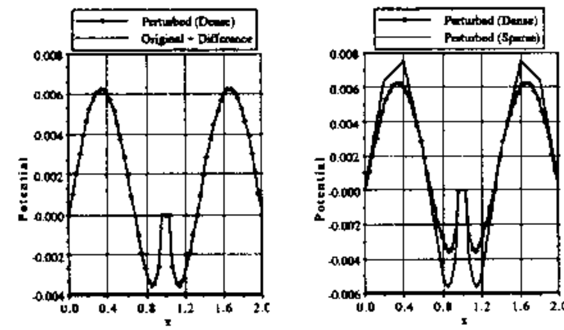
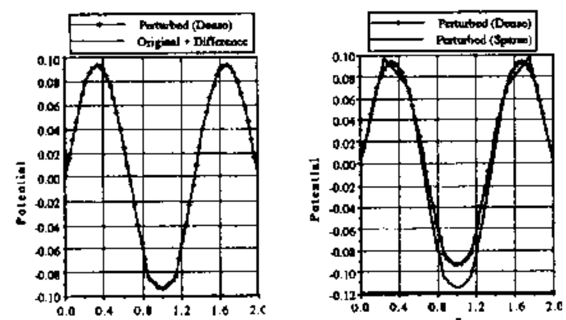


Figure 2. Geometry for dielectric obstacle



(a) (b)

Figures 3(a) and 3(b). Numerical results for the potential along the horizontal line abutting the bottom of the PEC obstacle



(a) (b)

Figures 4(a) and 4(b). Numerical results for the potential along the horizontal line through the center of the dielectric obstacle