Scattering from Chiral Cylinders of Circular Cross-Sections

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Abstract: A semi-analytical solution is presented to the problem of electromagnetic scattering from a collection of parallel chiral cylinder of a circular cross-section. The cylinders are illuminated by either a TE_z or a TM_z incident plane wave. The solution procedure is useful in predicting the scattering properties of arbitrary shaped two-dimensional scatterers composed of dielectric, conducting, and chiral media.

Introduction

Chiral media have been studied over the years for many applications involving antennas and arrays, antenna radomes, microstrip substrates and waveguides [1-6]. Coating with chiral material is also attempted for reducing radar cross-section of targets. Unlike dielectric or conducting cylinders, chiral scatterers produce both co-polarized and cross-polarized scattered fields which adds to the complexity of the analysis of multiple cylinders considered in this paper.

Formulation

Chiral medium is characterized by different phase velocities for right-hand circularly polarized waves (RCP) and left-hand circularly polarized waves (LCP) with two different bulk wave numbers k_+ and k_- which are given by

$$k_{\pm} = k[\sqrt{1+x^2} \pm x] \tag{1}$$

Where $k = \omega\sqrt{\mu\varepsilon}$ and $x = \sqrt{\frac{\mu}{\varepsilon}}\xi_c$, and ξ_c is the chiral admittance [1].

Considering an E-polarized incident wave (TM_z), the incident electric field of a plane wave is expressed in cylindrical coordinates system as

$$E_z^{inc}(\rho_i,\phi_i) = E_0 e^{jk_0\rho_i'\cos(\phi_i'-\phi_0)} e^{jk_0\rho_i\cos(\phi_i-\phi_0)}$$
$$= E_0 e^{jk_0\rho_i'\cos(\phi_i'-\phi_0)} \sum_{n=-\infty}^{\infty} J_n(k_0\rho_i) e^{jn\phi_i} \tag{2}$$

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where k_0 is the free space wave number and ϕ_0 is the angle of incidence of the plane wave with respect to the negative x axis. This is in terms of the cylindrical coordinates of the i^{th} cylinder, whose center is located at (ρ_i', ϕ_i'). The corresponding ϕ component of the magnetic field is given by

$$H_{\phi i}^{inc}(\rho_i, \phi_i) = \frac{E_0}{j\eta_0} e^{jk_0\rho_i' \cos(\phi_i' - \phi_0)} \sum_{n=-\infty}^{\infty} J_n'(k_0\rho_i) e^{jn\phi_i} \tag{3}$$

The resulting z component of the scattered electric field from a single cylinder and the transmitted z component of the field inside the chiral material of the i^{th} cylinder can be expressed, respectively, as

$$E_{zi}^{s}(\rho_i, \phi_i) = E_0 \sum_{n=-\infty}^{\infty} C_{in} H_n^{(2)}(k_0\rho_i) e^{jn\phi_i} \tag{4}$$

$$E_{zi}^{c}(\rho_i, \phi_i) = E_0 \sum_{n=-\infty}^{\infty} [A_{in} J_n(k_+\rho_i) + B_{in} J_n(k_-\rho_i)] e^{jn\phi_i} \tag{5}$$

while the corresponding ϕ components of the magnetic fields are obtained as,

$$H_{\phi i}^{s}(\rho_i, \phi_i) = \frac{1}{j\eta_0 k_0} \frac{\partial E_z^s(\rho_i, \phi_i)}{\partial \rho_i} = \frac{E_0}{j\eta_0} \sum_{n=-\infty}^{\infty} C_{in} H_n^{(2)'}(k_0\rho_i) e^{jn\phi_i} \tag{6}$$

$$H_{\phi i}^{c}(\rho_i, \phi_i) = \frac{1}{j\eta_{ci} k_i} \frac{\partial E_z^c(\rho_i, \phi_i)}{\partial \rho_i} = \frac{E_0}{j\eta_{ci}} \sum_{n=-\infty}^{\infty} \left[A_{in} J_n'(k_+\rho_i) + B_{in} J_n'(k_-\rho_i) \right] e^{jn\phi_i} \tag{7}$$

where $\eta_{ci} = \sqrt{\mu_i / \left[\varepsilon_i \left(1 + x^2\right)\right]}$.

The above expressions indicate that the incident field, the scattered, and the internal fields are based on the local coordinates (ρ_i, ϕ_i) of the i^{th} cylinder. However, the interaction between the cylinders in terms of multiple scattered fields will require a representation of the scattered field from one cylinder in terms of the local coordinates of another. Therefore, the addition theorem of Bessel and Hankel functions are used to transfer the scattered field components from one set of coordinates to another. As an example the scattered fields from the g^{th} cylinder in terms of the i^{th} cylinder are presented by

$$E_{zg}^{s}(\rho_i, \phi_i) = E_0 \sum_{n=-\infty}^{\infty} C_{gn} \sum_{m=-\infty}^{\infty} J_m(k_0\rho_i) H_{m-n}^{(2)}(k_0 d_{ig}) e^{j[m\phi_i - (m-n)\phi_{ig}]} \tag{8}$$

$$H^{s}_{\phi g}(\rho_i,\varphi_i)=\frac{E_0}{j\eta_0}\sum_{n=-\infty}^{\infty}C_{gn}\sum_{m=-\infty}^{\infty}J'_m(k_0\rho_i)H^{(2)}_{m-n}(k_0 d_{ig})e^{j[m\phi_i-(m-n)\phi_{ig}]} \quad (9)$$

The z component of the scattered magnetic field and the transmitted z component inside the chiral material of the i^{th} cylinder can be expressed as,

$$H^{s}_{zi}(\rho_i,\phi_i)=j\frac{E_0}{\eta_0}\sum_{n=-\infty}^{\infty}D_{in}H^{(2)}_n(k_0\rho_i)e^{jn\phi_i} \quad (10)$$

$$H^{c}_{zi}(\rho_i,\varphi_i)=j\frac{E_0}{\eta_{ci}}\sum_{n=-\infty}^{\infty}[A_{in}J_n(k_+\rho_i)-B_{in}J_n(k_-\rho_i)]e^{jn\phi_i} \quad (11)$$

while the corresponding ϕ components of the electric fields are as,

$$E^{s}_{\phi i}(\rho_i,\phi_i)=\frac{E_0}{j\eta_0 k_0}\frac{\partial H^{s}_z(\rho_i,\phi_i)}{\partial\rho_i}=E_0\sum_{n=-\infty}^{\infty}D_{in}H^{(2)'}_n(k_0\rho_i)e^{jn\phi_i} \quad (12)$$

$$E^{c}_{\phi i}(\rho_i,\phi_i)=\frac{E_0}{j\eta_{ci}k_i}\frac{\partial H^{c}_z(\rho_i,\phi_i)}{\partial\rho_i}=E_0\sum_{n=-\infty}^{\infty}[A_{in}J'_n(k_+\rho_i)-B_{in}J'_n(k_-\rho_i)]e^{jn\phi_i} \quad (13)$$

Representing these scattered field components from the g^{th} cylinder in terms of the local coordinates of the i^{th} is given by

$$H^{s}_{zg}(\rho_i,\phi_i)=j\frac{E_0}{\eta_0}\sum_{n=-\infty}^{\infty}D_{gn}\sum_{m=-\infty}^{\infty}J_m(k_0\rho_i)H^{(2)}_{m-n}(k_0 d_{ig})e^{j[m\phi_i-(m-n)\phi_{ig}]} \quad (14)$$

$$E^{s}_{\phi g}(\rho_i,\phi_i)=E_0\sum_{n=-\infty}^{\infty}D_{gn}\sum_{m=-\infty}^{\infty}J'_m(k_0\rho_i)H^{(2)}_{m-n}(k_0 d_{ig})e^{j[m\phi_i-(m-n)\phi_{ig}]} \quad (15)$$

The solution for the unknown coefficients A_{in}, C_{in}, B_{in} and D_{in} can be obtained by applying the appropriate boundary conditions on the surface of all cylinders. As an example, for the i^{th} cylinder, we have

$$E^{inc}_{zi}+\sum_{g=1}^{M}E^{s}_{zg}=E^{c}_{zi}\ ,\ H^{inc}_{\phi i}+\sum_{g=1}^{M}H^{s}_{\phi g}=H^{c}_{\phi i}\ ,\ \sum_{g=1}^{M}H^{s}_{zg}=H^{c}_{zi}\ ,\ \sum_{g=1}^{M}E^{s}_{\phi g}=E^{c}_{\phi i} \quad (16)$$

All boundary conditions on the surfaces of all cylinders yield a matrix equation, which is solved numerically after appropriate truncation.

Sample Numerical Results

Verification of the developed formulation is presented here in Figures 1 and 2, by comparing the numerical results of the echo width for a collection of parallel cylinders simulating a strip with the corresponding published results based on the method of moments technique presented in [2]. In these figures, our numerical results are presented by dashed lines in the co-polarized patterns and by solid lines in the cross-polarized patterns. Good agreements are observed.

$\mu_r = 2, \varepsilon_r = 3, \xi_c = 0.0005,$
frequency =300 MHz

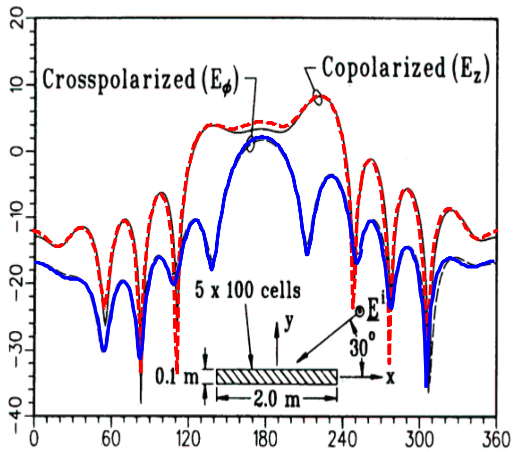

Figure 1. The co-polarized and cross-polarized bi-static echo width of a 0.1×2 m homogeneous chiral slab excited by a TM_Z plane wave incident at 30^0 off the x-axis (Figure 8 in [2]).

$\mu_r = 2, \varepsilon_r = 3, \xi_c = \pm 0.0005,$
frequency =300 MHz

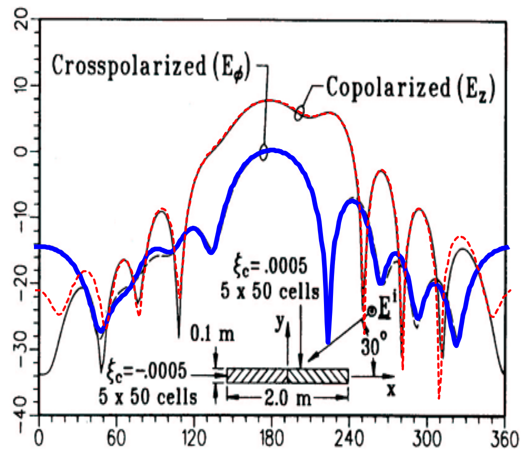

Figure 2. The co-polarized and cross-polarized bi-static echo width of a 0.1×2 m inhomogeneous chiral slab excited by a TM_Z plane wave incident at 30^0 off the x-axis (Figure 10 in [2]).

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