Analysis of Multilayer Frequency Selective Surfaces for Transmitarray Antenna Applications

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Abstract: Many transmitarray antennas are designed with multilayer frequency selective surface (M-FSS) type elements. The goal of this paper is to discover the theoretical limits of M-FSS for transmitarray designs, which is general for arbitrary element shapes. An analytical study on the transmission coefficient of multiple-conductor layers separated by dielectric materials has been carried out, and the transmission phase range has been determined according to the number of layers, substrate permittivity, and separation between conductor layers. Based on the analytical study, the best performance of the transmitarray antenna using frequency selective surfaces (FSS) is first determined by the selection of the number of layers, substrate material, and separation between conductor layers, even before the selection of the element shape. The effectiveness of the proposed approach has been validated through numerical simulations of several examples of FSS such as dipole and loop geometries.

Keywords: Frequency selective surfaces (FSS), Transmitarray antenna, Transmission phase range

1. Introduction

A transmitarray antenna consists of an illuminating feed source and a flat transmitting surface composed of one or multiple layers, as shown in Fig. 1. The feed source is usually located on an equivalent focal point. On the transmitting surface, there is an array of printed antenna elements. The transmission coefficients of these elements are individually designed to convert the spherical phase front from the feed to a planar phase front. As a result, a focused radiation beam can be achieved with a high gain. The frequency selective surfaces approach is popularly used to control the phase of each element in the array individually by varying the element's dimensions [1-3]. However, the required phase compensation for practical designs cannot be achieved by only one layer of the printed antenna elements array [1, 2]. Thus, multi-layer design in which the layers are separated by either air gap or dielectric material is required to increase the transmission phase range of the antenna element.

This paper organizes the procedures of designing transmitarray antennas using frequency selective surfaces. The transmission phase range limit is first determined according to the number of layers, the substrate material, and the separation between layers. The unit cell design becomes a secondary step where the element dimensions are to be optimized for the maximum transmission phase range possible.
2. Single Layer Analysis

A single layer with a conducting element can be considered as a two-port system [1-2], as shown in Fig. 2. It is assumed to be illuminated on both sides by a normally incident plane wave. The complex amplitude of the incident and reflected plane waves are $\vec{E}_1^+$ and $\vec{E}_1^-$, respectively, at the left side terminal plane. Similarly, $\vec{E}_2^+$ and $\vec{E}_2^-$ are the complex amplitude of the incident and reflected plane waves, respectively, at the right side terminal plane.

According to the linear two-port networks theory [4], these four complex waves are related to each other as,

$$\begin{bmatrix} E_1^+ \\ E_2^+ \\ E_1^- \\ E_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} E_1^- \\ E_2^- \end{bmatrix}$$

where $[S]$ is the scattering matrix of the two-port system. This layer is considered reciprocal [4],

$$S_{12} = S_{21}, \quad (1)$$

Furthermore, for lossless layer we have [4],

$$|S_{11}|^2 + |S_{21}|^2 = 1, \quad (2)$$
$$|S_{12}|^2 + |S_{22}|^2 = 1, \quad (3)$$
$$S_{11}S_{12}^* + S_{21}S_{22}^* = 0. \quad (4)$$

and due to symmetry,

$$S_{11} = S_{22}. \quad (5)$$

With equation (2), we can get

$$|S_{21}| = \sqrt{1 - |S_{11}|^2}. \quad (6)$$

By substituting equations (1), (5) and (6) in equation (4), we get,

$$|S_{11}|e^{i(\xi S_{11})} \sqrt{1 - |S_{11}|^2}e^{-i(\xi S_{12})} + \sqrt{1 - |S_{11}|^2}e^{i(\xi S_{12})} |S_{11}|e^{-i(\xi S_{11})} = 0$$
$$e^{i(\xi S_{11})}e^{-i(\xi S_{12})} + e^{i(\xi S_{12})}e^{-i(\xi S_{11})} = 0$$

Fig. 2. Single layer with a conducting element.
\[ \angle S_{11} = \angle S_{12} + \frac{\pi}{2}, \] (7)

Based on Fresnel reflection and transmission coefficients [5], while neglecting the higher order harmonics, we get,

\[ S_{12} = 1 + S_{11}. \] (8)

By substituting equation (7) in equation (8), we get,

\[
|S_{12}|e^{i\angle S_{12}} = 1 + |S_{11}|e^{i\left(\angle S_{12} + \frac{\pi}{2}\right)},
\]

\[
|S_{12}| - |S_{11}|e^{i\frac{\pi}{2}} = e^{-i\angle S_{12}},
\]

\[
|S_{12}| - j|S_{11}| = \cos(\angle S_{12}) - j\sin(\angle S_{12}). \] (9)

Equation (9) can be decomposed into two equations representing the real and imaginary parts, thus,

\[
|S_{12}| = \cos(\angle S_{12}) \quad \text{and} \quad |S_{11}| = \sin(\angle S_{12}). \] (10)

From equations (1), (5), (7) and (10), the S-matrix of a single layer of a transmitarray antenna is [1]:

\[
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} = 
\begin{bmatrix}
\sin(\angle S_{12}) e^{i\left(\angle S_{11} + \frac{\pi}{2}\right)} & \cos(\angle S_{12}) e^{i\angle S_{12}} \\
\cos(\angle S_{12}) e^{i\angle S_{12}} & \sin(\angle S_{12}) e^{i\left(\angle S_{11} + \frac{\pi}{2}\right)}
\end{bmatrix}. \] (11)

Thus, the transmission coefficient magnitude \(|S_{12}|\) is equal to the cosine of its phase \(\angle S_{12}\) regardless of the shape of the element. Figure 3 is a polar plot of \(S_{12}\), such that the magnitude represents \(|S_{12}|\) and the angle represents \(\angle S_{12}\), where \(|S_{12}| = \cos(\angle S_{12})\).

![Fig. 3. Transmission coefficient of a single layer configuration.](image)

The transmission phase magnitude relationship represents a circle on the polar plot. Thus, the maximum transmission coefficient (\(|S_{12}| = 1 = 0 \text{ dB}\)) is achieved only at multiples of \(2\pi\) (\(\angle S_{12} = 0^\circ, 360^\circ, ...\)). Also, we can determine the phase range for -1 dB and -3 dB transmission coefficient as shown in Fig. 3. The maximum phase range that can be achieved in a single layer transmitarray is \(54^\circ\) for -1 dB transmission coefficient and \(90^\circ\) for -3 dB transmission coefficient regardless of the shape of the conducting element. To achieve the maximum transmission phase range, we have to select a suitable element shape such that by varying its dimensions within the allowed periodicity of the array unit cell, the complete circle in the polar plot of Fig. 3 can be achieved.

To determine the accuracy of the above phase limits, two different single layer unit cells of a dipole and a double square loop elements are shown in Fig. 4 and are simulated separately at 8.4 GHz with half wavelength periodicity \(P = \lambda/2\) using Ansoft Designer software [6]. Figures 5(a) and 5(b) depict the transmission coefficient magnitude and phase of the elements with respect to the variation of their dimensions (the dipole length \(L\) and the double square loop side lengths \(L_1\) and \(L_2\) while keeping the separation \(S\), and the widths \(W\), \(W_1\) and \(W_2\) constant). The results shown in Fig. 5 confirm with the circle
obtained analytically in Fig. 3. The range of varying the dipole length (from \( L = 0 \) to \( L = P = \lambda_0/2 \approx 17.5 \) mm) is not sufficient to cover the phase range for a complete circle as shown in Fig. 5(a), while the double square loop element is capable of achieving the complete circle as shown in Fig. 5(b).

![Fig. 4. Unit cell of (a) a dipole element, and (b) a double square loop element [3].](image)

![Fig. 5. Transmission coefficient of the single layer (a) dipole element, (b) double square loop element.](image)

### 3. Multi Conductor Layers

Next we aim to obtain the S-matrix of multi-layer configurations as shown in Fig. 6 in order to determine the overall transmission coefficient \( S_{12} \). As a starting point, one develops the S-matrix of any two cascaded layers using the knowledge of the S-matrix of each individual layer as \([1-2]\),

\[
\begin{bmatrix}
S_{11}^c & S_{12}^c \\
S_{21}^c & S_{22}^c
\end{bmatrix} = \begin{bmatrix}
\frac{S_{11}^2S_{12}^1S_{21}^1 + S_{11}^1}{1 - S_{11}^2S_{22}^1} & \frac{S_{11}^1S_{22}^1}{1 - S_{11}^2S_{22}^1} \\
\frac{S_{21}^2S_{22}^1}{1 - S_{21}^2S_{22}^1} & \frac{S_{22}^2S_{22}^1S_{22}^1 + S_{22}^2}{1 - S_{22}^2S_{22}^1}
\end{bmatrix}
\]

(12)

where \( S_{11}^1, S_{12}^1, S_{21}^1, \) and \( S_{22}^1 \) are the S-parameters of the first layer, \( S_{11}^2, S_{12}^2, S_{21}^2, \) and \( S_{22}^2 \) are the S-parameters of the second layer, \( S_{11}^c, S_{12}^c, S_{21}^c, \) and \( S_{22}^c \) are the S-parameters of cascaded two layers. Accordingly, the S-matrix of multiple-conductor layers separated by dielectric substrate as shown in Fig. 6, can be computed (and hence the transmission coefficient \( S_{12} \)) by repeatedly cascading the S-matrices of the conductor layer defined in equation (11) and the S-matrix of the dielectric substrate defined as [7],

\[
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{\Gamma(1 - e^{-|\beta|L_a})}{1 - \Gamma^2e^{-|\beta|L_a}} & \frac{(1 - \Gamma^2)e^{-|\beta|L_a}}{1 - \Gamma^2e^{-|\beta|L_a}} \\
\frac{(1 - \Gamma^2)e^{-|\beta|L_a}}{1 - \Gamma^2e^{-|\beta|L_a}} & \frac{\Gamma(1 - e^{-|\beta|L_a})}{1 - \Gamma^2e^{-|\beta|L_a}}
\end{bmatrix}
\]

(13)

where,

\[
\Gamma = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} \quad \text{and} \quad \beta = \frac{2\pi\sqrt{\varepsilon_r}}{\lambda_0}.\]
The S-matrix of the dielectric substrate is a function of the dielectric permittivity $\varepsilon_r$ and the substrate thickness $L_d$, while the S-matrix of the conducting element layer is a function of its $\angle S_{12}$.

Figure 7(a-c) presents the relation between the transmission coefficient magnitude and phase of the double layer, triple layer, and quaternary layer configurations for different substrate permittivity with constant substrate electrical thickness ($\beta L_d = 90^\circ$). We conclude that the maximum transmission phase range of the double layer for -1dB and -3dB transmission coefficients are 170° and 228.5°, respectively. For the triple layer, the maximum transmission phase of 308° for -1dB transmission coefficient and a full phase range of 360° for -3dB transmission coefficient can be achieved. The quaternary layer can achieve full phase range of 360° for -1dB transmission coefficient.

Figure 8 presents the transmission coefficient of the dipole element in (a) double layer (b) triple layer (c) quaternary layer, and the double square loop element in (d) double layer (e) triple layer (f) quaternary layer.
Numerical analyses of different number of layers are carried out at 8.4 GHz with half wavelength periodicity ($P = \frac{\lambda_0}{2}$) using the dipole and the double square loop elements as shown in Fig. 8 using both CST Studio Suite software [8] and Ansoft HFSS [9]. The numerical simulations show small shift from the analytical results at some points when the permittivity increases. This is because the physical separation between layers decreases with the increase of the substrate permittivity for constant electrical length and leads to the increase of the high-order mode coupling between layers. Within the specified period, $P$, the range of varying the dipole element length is not sufficient to cover the maximum phase range as shown in Fig. 8(a-c), while the double square loop element is capable of achieving the maximum phase range as shown in Fig. 8(d-f). This illustrates the importance of selecting an element like the double square loop element rather than the dipole element in order to achieve the maximum phase range possible.

4. Conclusions

Analytical study on the transmission coefficient of multi-layer conductors separated by dielectric material is presented in this paper. The limits of the transmission phase range for -1dB and -3dB transmission coefficients have been derived according to the number of layers, substrate materials, and layer separations. These analytical limits are generally applicable, independently from the selection of a specific element shape. The proposed limits are validated through several numerical simulations. The condition of neglecting the high-order coupling effect between the conducting layers is considered in the analytical derivations.

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References