General Relativity HW3 Problems

1. Three events A, B, C are seen by an observer O to occur in the order ABC. Another Observer O’ sees the same three events occur in the order CBA. Is it possible that a third observer O” could see the events in the order ACB? Support your conclusions by drawing a spacetime diagram.

2. On a ct-x spacetime diagram, draw four events A, B, C and D such that A can cause B and C, B can cause D but not C, and C cannot cause D. Is such a situation possible in Galilean Relativity?

3. Prove that in special relativity $(\Lambda^0)^2 \geq 1$.

4. Consider objects $N_{ij}$ and $M^{ij}$ in 2D with components:

   \[
   N_{11} = a, N_{12} = b, N_{21} = c, N_{22} = d \\
   M^{11} = e, M^{12} = f, M^{21} = g, M^{22} = h
   \]

   Evaluate the following using index notation:
   a) $N_{ij}M^{ki}$
   b) $N_{ij}M^{kj}$
   c) $N_{ij}M^{ji}$
   d) $N_{ij}M^{ij}$

   For each of the above, rewrite and evaluate using matrix operations when possible.

Bonus problem:

Consider a Galilean boost from (ct, x, y, z) to $(ct', x', y', z')$ which corresponds to a boost along +x with a speed $v$. Find the matrix $\Lambda$ which enacts this transformation and then show that it satisfies $\Lambda^T g_1 \Lambda = g_1$ and $\Lambda \bar{g}_2 \Lambda^T = \bar{g}_2$ for $g_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and $\bar{g}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. 