General Relativity HW 7 Quiz Name________________

You know the drill!

1. A 2D surface is embedded into Minkowski space with metric $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$ by the following embedding function: $\{t, x, y, z\} = \{\sinh(u), \sin(v), cosh(u), \cos(v)\}$. Calculate the Riemann tensor for this 2D space in the $\{u, v\}$ coordinate system. You should be able to do all parts of this by hand!

$dt = \cosh(u)du$

$dx = \cos(v)dv$

$dy = \sinh(u)du$

$dz = -\sin(v)dv$

$$-dt^2 + dx^2 + dy^2 + dz^2 = -\cosh^2(u)du^2 + \cos^2(v)dv^2$$
$$+ \sinh^2(u)du^2 + \sin^2(v)dv^2$$
$$= -du^2 + dv^2 \Rightarrow g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Since this space is truly flat (globally constant metric) then $R^{\mu}{}_{\nu\rho\sigma} = 0$.

More love on the other side….so turn it over!!

2. Given:

$$g_{\mu\upsilon}=\begin{pmatrix} -1+r^2 & 0 & 0 & 0 \\ 0 & \frac{1}{1-r^2} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin^2(\theta) \end{pmatrix},\ R_{\mu\upsilon}=\begin{pmatrix} 3(-1+r^2) & 0 & 0 & 0 \\ 0 & \frac{3}{1-r^2} & 0 & 0 \\ 0 & 0 & 3r^2 & 0 \\ 0 & 0 & 0 & 3r^2 sin^2(\theta) \end{pmatrix}$$

find the form of $T_{\mu\upsilon}$ that would be the source for this geometry.

First note that $R_{\mu\nu} = 3g_{\mu\nu}$, then consider Einstein's Equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

Since $R = g^{\alpha\beta} R_{\alpha\beta}$ and $R_{\alpha\beta} = 3 g_{\alpha\beta}$ then $R = g^{\alpha\beta} 3 g_{\alpha\beta} = 12$

So we have:

$$3g_{\mu\nu} - 6 g_{\mu\nu} = 8\pi G T_{\mu\nu} \Rightarrow \boxed{T_{\mu\nu} = -\frac{3}{8\pi G} g_{\mu\nu}}$$

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