General Relativity HW9 Problems

1. Once across the event horizon of a Schwarzschild black hole of mass M, what is the longest proper time an observer can spend before reaching the singularity? Hint: You should ignore any attempts at angular motion and try to find an expression for $\frac{dr}{d\tau}$ where $d\tau^2 = -ds^2$.
2. Consider the spacetime specified by the line element

$$ds^2 = -(1-\frac{GM}{r})^2 dt^2 + (1-\frac{GM}{r})^{-2} dr^2 + r^2 d\Omega^2$$

a) Find a transformation to Eddington-Finkelstein-like coordinates (v,r,θ,ϕ) such that $g_{rr} = 0$ and show that the geometry is not singular at $r = GM$. You may start by looking for a new time coordinate $t = v + g(r)$ with $g(r)$ such that the g_{rr} term of the metric in the new coordinates is 0.

b) Sketch a plot analogous to our picture in class (EF for Schwarzschild) of the light cones in this geometry. That is take $ds^2 = 0$ with $d\Omega = 0$ and look for solutions from the metric.

3. Two observers in two rockets are hovering above a Schwarzschild black hole of mass *M*. They hover at a fixed radius *r* such that

$$(\frac{r}{2GM}-1)^{1/2} e^{r/4GM} = 1/2$$

with fixed angular position. The first observer leaves this position at *t = 0* and travels into the black hole **on a straight line path in a Kruskal diagram** until destroyed in the singularity at the point where the singularity crosses the line *R = 0* where *R* is the Kruskal radial coordinate (Note *r,t* are Schwarzschild coordinate values). The other observer continues to hover at radius *r*.

a) On a Kruskal diagram, sketch the worldlines of the two observers.

b) Is the observer who goes into the black hole following a timelike worldline?

4. Consider the Reissner-Nordstrom metric for a black hole with net charge Q:

$$ds^2 = -\left(1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\right)dt^2 + \left(1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 = -\Delta dt^2 + \Delta^{-1} dr^2 + r^2 d\Omega^2$$

a) Notice that the metric is badly behaved when $\Delta \to 0$ and when $\Delta \to \infty$. Find the values of r for which these occur.

b) Prove that $\Delta \to \infty$ represents a true curvature singularity by calculating the curvature related invariant $R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}$.

c) What is the energy-momentum tensor associated with this geometry? Compare this to the energy-momentum tensor associated to the Kerr geometry. Why are they so different? Hint: Use Mathematica for both of these!

d) What is the form of the metric for the “extremal” case when $G^2M^2 = GQ^2$? Notice anything?