Mathematical Methods in Physics HW1

1. Which of these form a group? If they do, identify which element acts as the identity. If they do not, specify which group criteria they do not meet.

- Integers with addition
- Integers with multiplication
- Rationals with addition
- Rationals with multiplication
- 3x3 matrices with arbitrary real elements with matrix multiplication
- 3x3 matrices with arbitrary real elements with addition
- Imaginary numbers with addition
- Imaginary numbers with multiplication

2. Which of these form a field? If they do then identify the field ingredients. If they do not, identify which ingredients go wrong.

- 2D rotations matrices with matrix addition and matrix multiplication
- 2D diagonal matrices with real elements with matrix addition and matrix multiplication
- 2D arbitrary with real elements with matrix addition and matrix multiplication

3. Which of these constitute a vector space? If they do, show that they do. If they don't, show why they don't.

- An n-tuple of complex numbers over the field of real numbers
- An n-tuple of imaginary numbers over the field of real numbers
- An n-tuple of imaginary numbers over the field of complex numbers
- An n-tuple of complex numbers over the field of imaginary numbers

4. Show that even and odd integers do not form a group under multiplication.

5. Consider $\mathbb{R}^3$ and a usual set of orthonormal basis vectors, $\hat{\imath}, \hat{\jmath}, \hat{k}$. Show that the set $\hat{a} = \hat{\imath}, \hat{b} = \hat{\jmath}, \hat{c} = \frac{1}{3}(\hat{\imath} + \hat{\jmath} + \hat{k})$ forms a basis, though not an orthonormal one (well at least it is normalized though not orthogonal).

6. Show that all groups with only three elements are isomorphic. How many different (non-isomorphic) variations of four element groups are there?

7. Prove that if the set of vectors $\{x_i\}$ is a basis for a vector space V, then for c_i arbitrary nonzero scalars, so is the set $\{c_i x_i\}$.