Math Methods HW 9 Quiz Name___________________

You can try both problems below, but you will only receive credit for the most correct solution.

1.(*10 pts*) Evaluate the following:

a) $I = \oint_C \frac{4z^4 - z}{z(z-1)(z-3)} dz$ around a circular contour centered at $z = 0$ and of radius $|z| = 2$.

The contour will only enclose $z = 0$ and $z = 1$, so we only need the residues coming from those singularities. Thus:

$$I = \oint_C \frac{4z^4 - z}{z(z-1)(z-3)} dz = \oint_{C_1} \frac{4z^4 - z/z(z-3)}{z-1} dz + \oint_{C_0} \frac{4z^4 - z/(z-1)(z-3)}{z} dz = 2\pi i \left[\frac{3}{-2} + \frac{0}{3}\right] = -3\pi i$$

b) $I = \oint_C \frac{4z^4 - z}{z(z-1)(z-3)} dz$ around a circular contour centered at $z = 4i$ and of radius $|z| = 2$.

The contour does not enclose any of the singular points so the integrand is analytic within and on the contour, hence $I = \oint_C \frac{4z^4 - z}{z(z-1)(z-3)} dz$=0

2. (*10 pts*) Give the Laurent expansions for:

a) $\frac{1}{e^z z^2}$

$$\frac{1}{e^z z^2} = \frac{e^{-z}}{z^2} = \frac{1}{z^2} \sum_0^\infty \frac{(-z)^n}{n!} = \sum_0^\infty \frac{1}{(-z)^2} \frac{(-z)^n}{n!} = \sum_0^\infty \frac{(-z)^{n-2}}{n!}$$

b) $\frac{z^2 - 2z + 1}{z^2 - z}$

$$\frac{z^2 - 2z + 1}{z^2 - z} = \frac{(z-1)(z-1)}{z(z-1)} = 1 - \frac{1}{z}$$