Particle Physics HW2

1. To get comfortable with manipulations in special relativity, we will start with a simple version that uses smaller vectors and matrices. Consider SO(1,1) which by our counting in class only has one free parameter. Following what we did in class, consider how it would act on a vector of coordinate differentials $dx^\mu = \begin{pmatrix} cdt \\ dx \end{pmatrix}$ where $\mu = 0,1$.

   a) Construct a matrix representing the one type of transformation in this group. Verify explicitly that it satisfies the conditions defining SO(1,1).

   b) Construct the dual vector dx_μ.

   c) Separately transform the vector and dual vector, i.e. determine $dx^{\mu'}$ and $dx_{\mu'}$, then confirm that $dx_\mu dx^\mu = dx_{\mu'} dx^{\mu'}$, i.e. that it is invariant.

2. We often write things as matrices because the rules of matrix multiplication quickly reproduce certain algorithms for combining many quantities. But matrices are kinda dumb. A basic matrix is just a square array of numbers. Often the things we are working with have more structure than just a simple array of numbers. In this problem you will learn what in the world I mean here. Consider the matrix $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$. For the next three questions you will consider how these components transform under an SO(2) transformation (the basic form of which you must first write down based on how it acts on vectors, e.g. $dx^\mu = \begin{pmatrix} dx \\ dy \end{pmatrix}$):

   a) If the matrix represents a (0,2)-tensor.

   b) If the matrix represents a (2,0)-tensor.

   c) If the matrix represents a (1,1)-tensor (be careful since it could represent two different (1,1)-tensors!)

3. One of the nice things about using superscripts and subscripts to indicate tensor properties is that we can look at a complicated object and immediately read off what sort of tensor it is, i.e. how it will transform under Lorentz transformations. For example $T_{\lambda\mu}{}^{\kappa\rho}{}_\nu$ is certainly ugly, but it is just a (2,3)-tensor meaning it will transform like $T_{\lambda'\mu'}{}^{\kappa'\rho'}{}_{\nu'} = \Lambda^\lambda{}_{\lambda'}\Lambda^\mu{}_{\mu'}\Lambda^\nu{}_{\nu'}\Lambda^{\kappa'}{}_\kappa \Lambda^{\rho'}{}_\rho T_{\lambda\mu}{}^{\kappa\rho}{}_\nu$. For each of the following, determine what type of (p,q)-tensor it is and write down the appropriate transformation law:

   a) $dx_\mu dx^\mu$

   b) $dx_\mu dx^\nu$

   c) $dx_\mu dx_\nu$

   d) $T^{\mu\nu} W^\rho{}_{\mu\lambda\kappa}$

   The following problem you do not have to do, but it is here for you to try. I will go over this one in lecture.

4. In this problem you will work through the kinematics of two of the most important "collision" type processes that we study to compare with experiment. Later in the course we will cover the

dynamics of these calculations (including forces), but for now you will explore what you can determine from purely kinematical reasoning.

a) Consider a scattering process where $A + B \rightarrow C + D + E + F$ where particle A begins with (relativistic) energy E_A and particle B begins at rest. Consider all particle masses known, i.e. $m_A, m_B, m_C, m_D, m_E, m_F$. Calculate the minimum energy E_A required for this process to occur in terms of $m_A, m_B, m_C, m_D, m_E, m_F$. You should be able to generalize your result to an arbitrary number of final particles.

b) Now consider a decay process wherein $A \rightarrow B + C$. Again consider all masses known, i.e. m_A, m_B, m_C. Determine the energies E_B and E_C in terms of the masses m_A, m_B, m_C. Also determine the magnitude of the outgoing 3-momentum of each decay product $|\vec{p}_B|$ and $|\vec{p}_C|$ in terms of the masses m_A, m_B, m_C.