1. Argue that any two groups with only two elements must be "the same" in the sense that their multiplication tables must have the same structure. It helps to remember the definition of a group.

2. Show that a 4 element group cannot be non-abelian.

3. Beginning with the generator $g_{R_{2y}}$ given in class, use the exponential map to show that this gives rise to the usual 3x3 matrix describing a rotation in the y-z plane in 3D (also given in class).

4. The generators of SU(3) can be written as $g_i = \frac{\lambda_i}{2}$ where:

$$
\begin{align*}
\lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},
\lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},
\lambda_6 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\lambda_7 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},
\lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.
\end{align*}
$$

Verify that these generators satisfy the algebra $[g_i, g_j] = if^{ijk}g_k$ where

$$
\begin{align*}
f^{123} &= 1, 
 f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2},
 f^{458} = f^{678} = \frac{\sqrt{3}}{2}
\end{align*}
$$

and the $f^{ijk}$ are totally antisymmetric in the three indices, i.e. $f^{ijk} = -f^{jik}$. If a particular index combination doesn’t appear in this list (or from cyclic permutations) it is 0. You need not verify all 28 versions of this relationship. A couple of examples should suffice for you to understand how this works. In particular checking $[g_4, g_5]$ would be the most instructive case.