Particle Physics HW3

1. Consider a decay process wherein $A \rightarrow B + C$. Again consider all masses known, i.e. m_A, m_B, m_C. Determine the energies E_B and E_C in terms of the masses m_A, m_B, m_C. Also determine the magnitude of the outgoing 3-momentum of each decay product $|\vec{p}_B|$ and $|\vec{p}_C|$ in terms of the masses m_A, m_B, m_C.
2. Using the method I showed you in class for finding elements of $F_{\mu\nu}$ by using the definition $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, find the F_{23} and F_{22} components.
3. Beginning with the generator $g_{R_{xy}}$ given in class for the 2D case, use the exponential map to and Taylor series to find the matrix transformation that this generates, that is find $R_{xy} = e^{ig_{R_{xy}}\theta}$ as a 2D matrix.
4. The generators of SU(3) can be written as $g_i = \frac{\lambda_i}{2}$ where: $\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$

   $$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

   Verify that these generators satisfy the algebra $[g_i, g_j] = if^{ijk}g_k$ where

   $$f^{123} = 1, \quad f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}$$

   and the f^{ijk} are totally antisymmetric in the three indices, i.e. $f^{ijk} = -f^{jik}$. If a particular index combination doesn't appear in this list (or from cylic permutations) it is 0. You need not verify all 28 versions of this relationship. A couple of examples should suffice for you to understand how this works. In particular checking $[g_4, g_5]$ would be the most instructive case.
5. Verify the algebra of the Lorentz generators, i.e. that $[J_i, J_j] = i\epsilon^{ijk}J_k$, $[K_i, K_j] = -i\epsilon^{ijk}J_k$ and $[J_i, K_j] = i\epsilon^{ijk}K_k$. You should try at least one nontrivial case for each, i.e. no repeated indices.
6. Verify the algebra of the linear combinations $J_{\pm i}$, i.e. that $[J_{\pm i}, J_{\pm j}] = i\epsilon^{ijk}J_{\pm k}$ and $[J_{+i}, J_{-j}] = 0$. Again, you should try at least one nontrivial combination from each case.