1. Starting from the Golden Rule for the decay rate for an arbitrary decay channel (not the simplified 2-body expression), argue that it gives \( \Gamma_i = 0 \) if the total mass of the decay products exceeds the mass of the decaying particle. We know this is true, but I want you to tease it out of the explicit form of the Golden Rule.

2. Consider \( A + A \rightarrow A + A \) in the \( ABC \) theory.
   a) Construct all of the lowest order diagrams for this process.
   b) Assuming \( m_B = m_C = 0 \), evaluate the total amplitude for the process to lowest order. Each contribution can be written in terms of an unevaluated integral over the internal momentum \( q \).
   c) Use your result and the Golden Rule to calculate \( \frac{d\sigma}{d\Omega} \) for this process to leading order.

Remember, this is a true 2-body scattering event.

3. Construct all of the diagrams that contribute to the decay of \( A \) in the \( ABC \) theory to the next order beyond what we did in class. No need to evaluate the diagrams, just draw them. You will quickly see why higher order sucks.

4. Consider an \( ABCD \) theory based on the Lagrangian

\[
\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi_{A} \partial^{\mu} \phi_{A} - \frac{1}{2} \left( \frac{m_A}{\hbar} \right)^2 \phi_{A}^2 + \frac{1}{2} \partial_{\mu} \phi_{B} \partial^{\mu} \phi_{B} - \frac{1}{2} \left( \frac{m_B}{\hbar} \right)^2 \phi_{B}^2 + \frac{1}{2} \partial_{\mu} \phi_{C} \partial^{\mu} \phi_{C} - \frac{1}{2} \left( \frac{m_C}{\hbar} \right)^2 \phi_{C}^2 \\
+ \frac{1}{2} \partial_{\mu} \phi_{D} \partial^{\mu} \phi_{D} - \frac{1}{2} \left( \frac{m_D}{\hbar} \right)^2 \phi_{D}^2 - g \phi_{A} \phi_{B} \phi_{C} - g \phi_{A} \phi_{B} \phi_{D}
\]

where \( m_A > m_B + m_C \) and \( m_D > m_A \).

Calculate the lifetime of \( A \) in this \( ABCD \) theory up to third order. This will involve identifying all possible decay products that can result from diagrams up to third order, constructing the diagrams and evaluating each one. You can leave the amplitude expressions in terms of one unevaluated integral over the internal momentum \( q \). Once you have your expressions, combine them (don’t forget the lower order result(s)) to obtain an overall expression for the lifetime.