Particle Physics HW 10 Quiz Name___KEY___

You know what to do I hope. Answer at least one and upload your response back to Canvas.

1) If the 3x3 CKM matrix was actually restricted to be special and unitary (as opposed to just unitary), would this allow us to make it real via redefinition of quark phases? Explain your reasoning.

For the unitary case we found that the unitary condition $U^{\dagger}U = I$ required 9 independent equations to be fulfilled by the 18 real parameters of the 3x3 complex matrix U, which removed 9 of the 18 leaving 9 free parameters. Applying the special condition in the unitary case removes one more free parameter leaving 8. However, for the real 3x3 matrix the orthogonality condition $U^{T}U = I$ provides 6 independent equations which remove 6 of the 9 parameters leaving 3 free parameters. Special in this case does not remove another parameter.

So from special unitarity we are left with 8 free parameters, while for special orthogonality we are left with 3. The quark phase and overall phase redefinitions can remove up to 4 of the free parameters, however 8-4=4 which is still one too many to declare the matrix real.

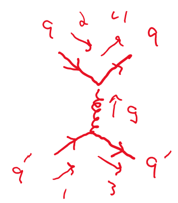
Turn over for second problem!!

2) For the state $|\mathbf{WTF}\rangle = \frac{1}{\sqrt{6}}(rr - bb + 2gg)$, determine the color factor for the lowest order $q + q' \rightarrow q + q'$ diagram. Here q and q' denote different quark flavors, so the only diagram that contributes is the one that we discussed in lecture and the one that you worked with in your homework.

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The hard way:



$\Rightarrow$ color factor is: $f = \frac{1}{4} c_3^\dagger \lambda^\alpha c_1 c_2^\dagger \lambda^\alpha c_4$

where: $c_1 c_2 = \frac{1}{\sqrt{6}}(rr + bb - 2gg) = c_3 c_4$

then: $= \frac{1}{4\sqrt{6}}\Big[c_3^\dagger \lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix} c_4^\dagger \lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix} + c_3^\dagger \lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix} c_4^\dagger \lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}$

$- 2 c_3^\dagger \lambda^\alpha \begin{pmatrix}0\\0\\1\end{pmatrix} c_4^\dagger \lambda^\alpha \begin{pmatrix}0\\0\\1\end{pmatrix}\Big]$

$= \frac{1}{24}\Big[(1\,0\,0)\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}(1\,0\,0)\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix} + (0\,1\,0)\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}(0\,1\,0)\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}$

$- 2(0\,0\,1)\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}(0\,0\,1)\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix} + (1\,0\,0)\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}(1\,0\,0)\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}$

$+ (0\,1\,0)\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}(0\,1\,0)\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix} - 2(0\,0\,1)\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}(0\,0\,1)\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}$

$- 2(1\,0\,0)\lambda^\alpha \begin{pmatrix}0\\0\\1\end{pmatrix}(1\,0\,0)\lambda^\alpha \begin{pmatrix}0\\0\\1\end{pmatrix} - 2(0\,1\,0)\lambda^\alpha \begin{pmatrix}0\\0\\1\end{pmatrix}(0\,1\,0)\lambda^\alpha \begin{pmatrix}0\\0\\1\end{pmatrix}$

$+ 4(0\,0\,1)\lambda^\alpha \begin{pmatrix}0\\0\\1\end{pmatrix}(0\,0\,1)\lambda^\alpha \begin{pmatrix}0\\0\\1\end{pmatrix}\Big]$

$= \frac{1}{24}\Big[\lambda_{11}^\alpha \lambda_{11}^\alpha + \lambda_{21}^\alpha \lambda_{21}^\alpha - 2\lambda_{31}^\alpha \lambda_{31}^\alpha + \lambda_{12}^\alpha \lambda_{12}^\alpha + \lambda_{22}^\alpha \lambda_{22}^\alpha$

$- 2\lambda_{32}^\alpha \lambda_{32}^\alpha - 2\lambda_{13}^\alpha \lambda_{13}^\alpha - 2\lambda_{23}^\alpha \lambda_{23}^\alpha + 4\lambda_{33}^\alpha \lambda_{33}^\alpha\Big]$

Since $\lambda_{21}\lambda_{21} = \lambda_{12}\lambda_{12}$, etc. $= \frac{1}{24}\Big[\lambda_{11}^\alpha \lambda_{11}^\alpha + 2\lambda_{12}^\alpha \lambda_{12}^\alpha - 4\lambda_{13}^\alpha \lambda_{13}^\alpha - 4\lambda_{23}^\alpha \lambda_{23}^\alpha$

$+ \lambda_{22}^\alpha \lambda_{22}^\alpha + 4\lambda_{33}^\alpha \lambda_{33}^\alpha\Big]$

$= \frac{1}{24}\Big[1 + \frac{1}{3} + 2 - 2 - 4 + 4 - 4 + 4 + 1 + \frac{1}{3} + \frac{16}{3}\Big]$

$= \frac{1}{3}$

The easy way: This is a superposition of sextet states, all of which have color factor $f = \frac{1}{3}$. So $f = \frac{1}{3}$ for $|WTF\rangle$!