You can try both problems below, but you will only receive credit for the most correct solution.

1. (10pts) In a 1+2D spacetime (based on SO(1,2) invariance) you have a dual vector with components $V_{\mu} = (1 \ 1 \ 1)$ and a (0,2)-tensor with components $M_{\mu \rho} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ when evaluated in a certain reference frame. Find the components of $V^{\mu} M_{\mu \rho}$, after a transformation described by $\Lambda^{\nu}_{\mu} = \begin{pmatrix} \gamma & 0 & \beta \gamma \\ -\beta \gamma & 0 & -\gamma \\ 0 & 1 & 0 \end{pmatrix}$. It may help to know that the inverse of the transformation matrix is $\begin{pmatrix} \gamma & \beta \gamma & 0 \\ 0 & 0 & 1 \\ -\beta \gamma & -\gamma & 0 \end{pmatrix}$.

First we form: $V^{\nu} = g^{\nu \nu} V_{\nu} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

Then: $M_{\nu \mu} V^{\nu} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} = W_{\nu}$

Then: $W_{\nu} \rightarrow W_{\nu}^{' \nu} = \Lambda_{\nu}^{\nu'} W_{\nu} = (\Lambda_{\nu}^{\nu'})^{-1} W_{\nu}$

\[
\begin{pmatrix} \gamma & 0 & -\delta \gamma \\ \delta \gamma & 0 & -\gamma \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma - \delta \gamma \\ \delta \gamma - \delta \\ -\gamma \end{pmatrix}
\]
2. (10pts) Consider the scattering event $A + A \rightarrow B + B$ where in the lab frame the incoming particles are moving at $90^\circ$ to each other and have equal mass $m_A$. Afterwards the two outgoing particles have equal mass $m_B$. If the incoming particles each have an energy $E_A$ in the lab frame, determine the energies of the outgoing particles in the center of momentum frame in terms of $m_A, m_B, E_A$.

In the lab frame: $p^A_A = \left( \frac{E_A}{c} \right) \hat{p}_A$, $p^A_{A'} = \left( \frac{E_A}{c} \right) \hat{p}_{A'}$ where $\hat{p}_A \cdot \hat{p}_{A'} = 0$.

In the c.o.m. frame: $p^h_A = \left( \frac{E_A}{c} \right) \hat{p}_A$, $p^h_{A'} = \left( \frac{E_A}{c} \right) \hat{p}_{A'}$ so $p^h_A + p^h_{A'} = \left( \frac{2E_A}{c} \right) \hat{0}$.

So conservation of 4-momentum $p^A_A + p^h_A = p^B_B + p^h_B$, leads to:

$$(p^A_A + p^h_A - p^B_B - p^h_B)(p^A_{A'} + p^h_{A'} - p^B_{B'} - p^h_{B'}) = 0.$$  

$p^h_A + p^h_{A'} + p^A_A + p^A_{A'} = -4 \frac{E_A}{c^2}$.

Then: $E_B = \sqrt{\frac{1}{2} m_B \frac{1}{c^2}} + \frac{1}{2} m_A^{1/2} + E_A^{1/2}$.