Particle Physics HW 2 Quiz

Name Key

You can try both problems below, but you will only receive credit for the most correct solution.

1. (*10pts*) In a 1+2D spacetime (based on SO(1,2) invariance) you have a vector with components $V^\mu = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and a (1,1)-tensor with components $M^\rho{}_\mu = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{pmatrix}$ when evaluated in a certain reference frame. Find the components of $M^{\rho'}{}_{\mu'} V^{\mu'}$ after a transformation described by $\Lambda^{\nu'}{}_\nu = \begin{pmatrix} \gamma & 0 & \beta\gamma \\ -\beta\gamma & 0 & -\gamma \\ 0 & 1 & 0 \end{pmatrix}$.

First of all $M^\rho{}_\mu V^\mu = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix} \equiv A^\mu$

and it transforms like a vector, i.e. $A^\mu \to A^{\mu'} = \Lambda^{\mu'}{}_\mu A^\mu$

hence $A^{\mu'} = \begin{pmatrix} \gamma & 0 & \beta\gamma \\ -\beta\gamma & 0 & -\gamma \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix} = \begin{pmatrix} 3\gamma - 3\beta\gamma \\ -3\beta\gamma + 3\gamma \\ 3 \end{pmatrix}$

Turn over for second problem!!

2. (*10pts*) Using the 2D rotation matrix $\Lambda^{i'}{}_i = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ as it acts on a displacement vector $dx^i = \begin{pmatrix} dx \\ dy \end{pmatrix}$, show explicitly that the combination $dx^i dx_i$ is invariant, i.e. $dx^i dx_i = dx^{i'} dx_{i'}$, by transforming dx^i and dx_i separately, and then combining the results to show that what you get is the same as what you started with.

$$dx^i \to dx^{i'} = \Lambda^{i'}{}_i dx^i = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} \cos\theta dx + \sin\theta dy \\ -\sin\theta dx + \cos\theta dy \end{pmatrix}$$

$$dx_i \to dx_{i'} = \Lambda^i{}_{i'} dx_i = dx_i \Lambda^i{}_{i'} = (dx \ dy)\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$= (dx\cos\theta + dy\sin\theta, \ -dx\sin\theta + dy\cos\theta)$$

$$dx^i dx_i \to dx^{i'} dx_{i'} = dx_{i'} dx^{i'}$$

$$= (dx\cos\theta + dy\sin\theta, \ -dx\sin\theta + dy\cos\theta)\begin{pmatrix} \cos\theta dx + \sin\theta dy \\ -\sin\theta dx + \cos\theta dy \end{pmatrix}$$

$$= \cos^2\theta dx^2 + 2\sin\theta\cos\theta dx dy + \sin^2\theta dy^2$$
$$+ \sin^2\theta dx^2 - 2\sin\theta\cos\theta dx dy + \cos^2\theta dy^2$$

$$= (\cos^2\theta + \sin^2\theta) dx^2 + (\sin^2\theta + \cos^2\theta) dy^2$$

$$= dx^2 + dy^2$$