Particle Physics HW 5 Quiz Name KEY

You can try both problems below, but you will only receive credit for the most correct solution.

1. **a**) (*5 points*) Determine an expression for the positive helicity form of $\psi^{(1)}$ for a Dirac fermion moving along the positive x-axis, i.e. with zero momentum along the y and z axes.

We can use the result from the HW by setting $p_y = p_z = 0$ and $p_x = p$. Then:

$$P_+ \psi^{(1)} = \frac{1}{2} A e^{i \frac{p_\mu x^\mu}{\hbar}} \frac{1}{p\hbar c} \begin{bmatrix} \left(\frac{E}{c} - p\right) p \\ \left(\frac{E}{c} - p\right) p \\ p\hbar c \\ p\hbar c \end{bmatrix}$$

$$= \frac{1}{2} A e^{i \frac{p_\mu x^\mu}{\hbar}} \frac{1}{\hbar c} \begin{pmatrix} \left(\frac{E}{c} - p\right) \\ \left(\frac{E}{c} - p\right) \\ \hbar c \\ \hbar c \end{pmatrix} .$$

b) (*5 points*) Show explicitly that your result from part (a) is **not** an eigenstate of S_y.

$$S_y = \frac{i\hbar}{2} \begin{pmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & 0 & -1 \\ & & 1 & 0 \end{pmatrix}$$

All of the factors multiplying both S_y and $P_+\psi^{(1)}$ will not play a role in determining if it is an eigenstate so ignoring them:

$$S_y P_+ \psi^{(1)} \sim \begin{pmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & 0 & -1 \\ & & 1 & 0 \end{pmatrix} \begin{pmatrix} \left(\frac{E}{c} - p\right) \\ \left(\frac{E}{c} - p\right) \\ \hbar c \\ \hbar c \end{pmatrix} = \begin{pmatrix} -\left(\frac{E}{c} - p\right) \\ \left(\frac{E}{c} - p\right) \\ -\hbar c \\ \hbar c \end{pmatrix} \underbrace{\neq K P_+ \psi^{(1)}}$$

So $P_+\psi^{(1)}$ is not an eigenstate.

Turn over for second problem!!

2) (*10 points*) Consider the Proca equation for a massive particle $\partial_\mu F^{\mu\nu} - \left(\frac{mc}{\hbar}\right)^2 A^\nu = 0$. Show that for a plane-wave solution of the form $A^\nu = Ae^{\frac{i}{\hbar}P_\mu x^\mu}\epsilon^\nu$, the four-momentum P^μ must be orthogonal to the polarization vector ϵ^μ in the 4D sense. (Yes this is the same result you proved in the HW for the massless case, but it has different implications in the massive case!) It may help to remember the relativistic energy-momentum-mass condition $P^\mu P_\mu = -\frac{E}{c^2}^2 + p^2 = -m^2c^2$.

$$\partial_\mu F^{\mu\nu} - \left(\frac{mc}{\hbar}\right)^2 A^\nu = 0$$

$$\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) - \left(\frac{mc}{\hbar}\right)^2 A^\nu = 0$$

Going to momentum space w/ $\partial_\mu \to \frac{i}{\hbar} P_\mu$ and $\partial^\mu \to \frac{i}{\hbar} P^\mu$ this becomes:

$$-\frac{1}{\hbar^2} P_\mu (P^\mu A^\nu - P^\nu A^\mu) - \left(\frac{mc}{\hbar}\right)^2 A^\nu = 0$$

$$-\frac{P_\mu P^\mu}{\hbar^2} A^\nu + \frac{1}{\hbar^2} P^\nu P_\mu A^\mu - \left(\frac{mc}{\hbar}\right)^2 A^\nu = 0$$

But $P_\mu P^\mu = -m^2c^2$

$$\cancel{\left(\frac{mc}{\hbar}\right)^2 A^\nu} + \frac{1}{\hbar^2} P^\nu P_\mu A^\mu - \cancel{\left(\frac{mc}{\hbar}\right)^2 A^\nu} = 0$$

Since P^ν is arbitrary we must have $P_\mu A^\mu = 0$.