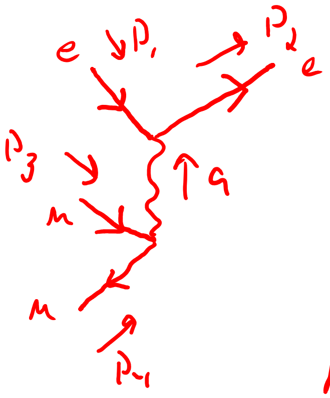
Particle Physics HW 9 Quiz Name KEY

You can try both problems below, but you will only receive credit for the most correct solution.

1. Consider the process $e + \mu + \mu^+ \to e$. Compute the expression for $\langle |M|^2 \rangle$ to lowest order leaving it in terms of traces.



$$\int \bar{u}(2)\, i g_e \gamma^\mu u(1)\, \bar{v}(4)\, i g_e \gamma^\nu u(3) \frac{-i\eta_{\mu\nu}}{q^2} (2\pi)^4 \delta^4(p_1 + q - p_2)(2\pi)^4 \delta^4(p_3 + p_4 - q) \frac{d^4 q}{(2\pi)^4}$$

$$M = -\bar{u}(2)\, i g_e \gamma^\mu u(1)\, \bar{v}(4)\, i g \gamma^\nu u(3) \frac{\eta_{\mu\nu}}{(p_3 + p_4)^2}$$

$$|M|^2 = \frac{\eta_{\mu\nu}\eta_{\alpha\beta}}{(p_3+p_4)^4} \bar{u}(2)\, i g_e \gamma^\mu u(1) \left[\bar{u}(2)\, i g_e \gamma^\alpha u(1)\right]^*$$

$$\times\, \bar{v}(4)\, i g \gamma^\nu u(3) \left[\bar{v}(4)\, i g \gamma^\beta u(3)\right]^*$$

$$\langle |M|^2 \rangle = \frac{1}{8} \frac{\eta_{\mu\nu}\eta_{\alpha\beta}}{(p_3+p_4)^4} g_e^4\, \mathrm{Tr}\left[\gamma^\mu (\not{p}_1 + m_e c) \gamma^\alpha (\not{p}_2 + m_e c)\right]$$

$$\mathrm{Tr}\left[\gamma^\nu (\not{p}_3 + m_\mu c) \gamma^\beta (\not{p}_4 - m_\mu c)\right]$$

Turn over for second problem!!

2) Is the singlet state of color $|9\rangle = \frac{1}{\sqrt{3}}\left(r\bar{r} + b\bar{b} + g\bar{g}\right)$ invariant under the transformation $U = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$? How do you know?

Well, first of all $|9\rangle$ is invariant under $SU(3)$ transformations of QCD. But $U^\dagger U = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} \neq I$ so $U \notin SU(3)$.

$\det U \neq 1$ as well.

Alternatively,

$$|9\rangle \to |9'\rangle = \frac{1}{\sqrt{3}}\left[\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right]$$

$$= \frac{1}{\sqrt{3}}\left[\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right] \neq |9\rangle$$