Lecture 17 – The Higgs Mechanism and Spontaneous Symmetry Breaking

"Broken" Symmetry

It is possible for a solution to the equations of motion to exhibit a symmetry of the original action. Consider:

\[ V(\phi) = -\frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \]

Then:

\[ L = \frac{\partial \phi}{\partial \tau} \cdot \frac{\partial \phi}{\partial \tau} - V(\phi) = \frac{\partial \phi}{\partial \tau} \cdot \frac{\partial \phi}{\partial \tau} + \frac{1}{2} m^2 \phi^2 - \frac{1}{4} \lambda \phi^4 = 0 \]

If we look for static solutions, i.e., \( \frac{\partial \phi}{\partial \tau} = 0 \) when

we want \( \frac{\partial \phi^2}{\partial \tau^2} = -m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 = 0 \) \( \Rightarrow \phi = \begin{cases} +1 & \text{not symmetric} \\ -1 & \text{not symmetric} \end{cases} \)

Notice this is even more explicit when we focus on small fluctuations (as we do when we study particle-like behavior of the underlying fields in the SM).

Consider \( \phi(x) = \phi_0 + \delta \phi(x) \) where \( \phi_0 \) is one of the static solutions above. To determine the equation of motion for the fluctuations \( \delta \phi(x) \), we simply substitute this back into the original Lagrangian:

\[ L(\delta \phi) = \frac{\partial}{\partial \tau} (\phi_0 + \delta \phi)(\frac{\partial}{\partial \tau} (\phi_0 + \delta \phi) - (\phi_0 + \delta \phi)^2 + (\phi_0 + \delta \phi)^4 \]

\[ = \frac{\partial}{\partial \tau} \frac{\partial}{\partial \tau} (\phi_0 + \delta \phi) \delta \phi + 2 \frac{\partial}{\partial \tau} \frac{\partial}{\partial \tau} (\phi_0 + \delta \phi) \delta \phi + \frac{\partial}{\partial \tau} (\phi_0 + \delta \phi) \delta \phi + \frac{\partial}{\partial \tau} (\phi_0 + \delta \phi) \delta \phi - (\phi_0 + \delta \phi)^2 + (\phi_0 + \delta \phi)^4 \]

\[ L_0 = \frac{\partial}{\partial \tau} \frac{\partial}{\partial \tau} (\phi_0 + \delta \phi) \delta \phi \quad \text{same as original} \]

\[ L_{-1} = \frac{\partial}{\partial \tau} \frac{\partial}{\partial \tau} (\phi_0 + \delta \phi) + (\phi_0 + \delta \phi)^2 \quad \text{L for } \phi \text{ has no symmetry breaking} \]

\[ L_{+1} = \frac{\partial}{\partial \tau} \frac{\partial}{\partial \tau} (\phi_0 + \delta \phi) + (\phi_0 + \delta \phi)^4 \quad \text{L for } \phi \text{ has symmetry breaking} \]

It is important to realize that the full underlying potential in both cases is symmetric. It’s just that when we focus on fluctuations about a particular solution that the symmetry is not realized, i.e., it appears to be broken.
Recall that for the complex scalar Higgs we had:

\[ \mathcal{L}(\phi, \bar{A}_\mu, A) = \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi) - \frac{1}{2} \lambda \phi^* \phi + \frac{1}{2} \lambda \phi^* \phi^2 + \frac{1}{16} F_{\mu \nu} F^{\mu \nu} + \text{massive vector} \]

This has a symmetric solution: \( \phi = 0, A = 0 \) where the \( \mathcal{L}(\phi, \bar{A}_\mu, A) \) looks just like the expression above.

We also considered the more interesting solution: \( \phi = \phi_0 = 0 \) where the Lagrangian for fluctuations \( \phi \phi^* \) becomes:

\( \phi_0 = 0 \)

\[ A = 0 \]

\[ \mathcal{L}(\phi, \bar{A}_\mu, A) = \frac{1}{2} \partial^\mu \phi(\partial_\mu \phi) + \frac{1}{2} \partial^\mu \phi^* \partial_\mu \phi^* + \frac{1}{16} F_{\mu \nu} F^{\mu \nu} + \text{massive vector} \]

We have done here is pretty much like what we just discussed for symmetry breaking with the difference that here we are breaking a continuous symmetry (i.e., \( \mathcal{L}(\phi, \bar{A}_\mu, A) \)) where before the symmetry was discrete, \( \phi \to -\phi \).

A picture will help...
The Higgs Potential: 
\[ \mathcal{L}_{\text{Higgs}} = -\frac{1}{2} \phi \cdot \phi + \frac{1}{4} \lambda (\phi \cdot \phi)^2 \]

\[ \phi = 0 \text{ solution} \]

The mass of a fluctuation can be associated with the coefficient of the quadratic term, 
\[ m^2 = \frac{\partial^2 \mathcal{L}}{\partial \phi \partial \phi} = \frac{\partial}{\partial \phi} \left( \frac{1}{2} \phi \cdot \phi \right) = \lambda \phi^2 \]

But the 2nd derivative is just expressing concavity!

Notes:
- 2nd derivative $> 0$ for $\phi$
- $= 0$ for $\beta$

What happened to original $\phi \to e^{i \beta}$?

The original symmetry is now encoded by "shifts" in $\beta$, i.e., $\beta \to \beta + \delta \beta$.

In words: The Higgs Mechanism" gives mass to the gauge fields of a "spontaneously broken" gauge symmetry through the coupling to an extra Higgs field $\phi$ (which of course has its own particles!).

The 2-polarization massless spin-1 gauge field "cants" the spin-$1/2\,\text{Goldstone\,boson}$ to get its 3rd polarization effect, which is required when $\frac{\partial^2 \mathcal{L}}{\partial \phi \partial \phi} = 0$.

This simple U(1) example can be generalized to the breaking of $SU(3) \times U(1)$, which is then explained by the mass of the $W^+, W^-$ bosons.

Furthermore, by coupling the fundamental matter fermions (taken to be massless for $SU(3)$ invariant) we can generate effective masses for these as well.

This still leaves the question: "How did the Higgs field ever get into the unstable solution in the first place?"
Suppose that the Higgs potential itself has “evolved” over the history of the universe.

\[ U(M) \]

\[ \Rightarrow \]

\[ U(M) \]

\[ \Rightarrow \]

\[ \Rightarrow \]

Lowest energy solution is symmetric \( SU(2) \times U(1)_Y \)

Still symmetric \( SU(2) \times U(1)_Y \)

Lowest energy solutions have broken symmetry \( U(1)_{EM} \)

How does this happen? Recall that as the universe ages, it expands and cools, hence the average energy density is decreasing.

So we can consider:

\[ \text{Energy} \]

One of the important things we will discover when we start doing calculations is that the “constant” coefficients in our Lagrangian actually change with the energy scale. But constants like \( g \) and \( \lambda \) are what determine the shape of the Higgs potential!

This might all sound strange, but you are probably already familiar with an example of this...
Consider a solid of magnetic dipoles at very high temperature. In this case the thermal motion is so extreme, that it overcomes the dipole-dipole interaction and everything looks random.

\[
\text{High } T : \quad \uparrow \quad \downarrow \\
\uparrow \quad \downarrow \\
\downarrow \quad \uparrow
\]

As we cool this system, the thermal motion eventually slows and is overtaken by the dipoles tending to align.

\[
\text{Low } T : \quad \uparrow \quad \downarrow \\
\uparrow \quad \downarrow \\
\downarrow \quad \uparrow
\]

What is perhaps counter-intuitive is that the high \( T \) state is actually more symmetric! It has \( SO(3) \) invariance. As low \( T \) there is a preferred axis in space, leaving only \( SO(2) \) invariance. So \( SO(2) \) is spontaneously broken to \( SO(2) \) in this system.

All of this can be modelled in terms of an "effective potential" for the dipole alignment in perfect analogy to the Higgs.

Note: The final preferred axis is completely undetermined in this case, hence the name spontaneous symmetry breaking.