Hierarchies of interaction:

- Neutrinos: Weak
- Charged leptons: EM, Weak
- Quarks: QCD, EM, Weak

```
"direct" interactions but...
```

In a given process, it must include all contributions:

\[
e^+ e^- \rightarrow \gamma + \gamma^* + q \bar{q} + \ldots
\]

Virtual states can bring in additional interactions!

Our next step will be to discuss QCD which means quarks; but we must also understand how QED plays out for quarks. We'll save the Weak interactions for last!
Lecture 27 - QCD The Force Awakens Page 2

**Quarks + QED**

Quarks can replace $e^+_q, n_q, u_q^-$ in any QED diagram w/ appropriate charge of $q = e \sqrt{\frac{4}{3}}$ (vertex factors).

Everything else same: $u_n, d_n, s_n, \bar{u}_n, \bar{d}_n, \bar{s}_n$ external states

spin-1 quarks: $\bar{s}_8$

propagators same, etc. (Note: color just along for ride)

**Quarks + QCD**

Color ($1, 2, 3$) plays role of "charges" and interactions via $8$ gluons.

3 gluon instead of 1 in QED!

Note in QED only one $Y$.

One practical complication is that QCD is defined in terms of quarks, but we only observe (and experiment) hadrons.

In fact since we often work with mesons ($\pi\pi$) and baryons ($\Lambda\Sigma$), it is useful to work with a linearly independent set of 8 basis states:

- "Ood" $17 = \frac{1}{\sqrt{2}}(\bar{u}_8 + \bar{t}_8)$
- $18 = \frac{1}{\sqrt{2}}(\bar{u}_8 - \bar{t}_8)$
- $19 = \bar{t}_8$
- $20 = \bar{t}_8$
- $21 = \frac{1}{\sqrt{2}}(\bar{t}_8 + \bar{u}_8)$
- $22 = \frac{1}{\sqrt{2}}(\bar{t}_8 - \bar{u}_8)$

`````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````````
Now that we have the field strength $F_{\mu \nu}$, we add the gauge invariant term:

$$
L_a = \frac{1}{16\pi} F_{\mu \nu}^a F^{\mu \nu}_a = \frac{1}{16\pi} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c)(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c)
$$

$$
= \frac{1}{16\pi} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) - \frac{1}{8\pi} f^{abc} A_\mu^b A_\nu^c (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) \\
$$

usual kinetic term

$$
- \frac{1}{8\pi} f^{abc} A_\mu^b A_\nu^c (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) \\
+ \frac{3}{16\pi} f^{abc} f^{def} A_\mu^b A_\nu^c A_\rho^d A_\sigma^e A_\tau^f \\
$$

These are gluon-gluon interactions!

Note that the gluon-gluon interactions critically depend on SU(3) being non-abelian, i.e. $f^{abc}$ is non-zero.

This is of course why photons in (abelian U(1)) EM do not interact with each other (at least classically).

These gluon-gluon interactions bring in a host of new effects including glueballs, confined, etc.
A useful way to think about quark-gluon interactions is as follows.

Consider \( q = (\begin{pmatrix} q_r \\ q_b \\ q_g \end{pmatrix}) \) and an associated \( \lambda' = \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \) [again from HW3]

\[
\bar{\pi}_{\lambda'}: \bar{\pi} \cdot \lambda' \cdot q = (\bar{\pi} \cdot \bar{q}_b \cdot \bar{q}_g) \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{pmatrix} \bar{q}_r \\ \bar{q}_b \\ \bar{q}_g \end{pmatrix} \right) = \bar{\pi} \cdot \bar{q}_b + \bar{\pi} \cdot \bar{q}_r + \bar{\pi} \cdot \bar{q}_g
\]

\[
\frac{1}{8i} (\bar{b} \bar{c} + \bar{c} \bar{b}) \quad \bar{c} \bar{b} = b \quad \bar{b} \bar{c} = c
\]

So the gluons are bi-colored (cC) while the quarks are just colored (r,b,g), anti-quarks (\( \bar{r}, \bar{b}, \bar{g} \)).

This will be immensely helpful in constructing Feynman diagrams for QCD.
Left compose:

<table>
<thead>
<tr>
<th>External State Labels</th>
<th>Internal Propagators</th>
<th>Vertex Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ABC$</td>
<td>$\frac{1}{q^2-m^2}$</td>
<td>$\bar{A} \gamma_\mu c - i\gamma_\mu$</td>
</tr>
<tr>
<td>$\text{QED}$</td>
<td>$\bar{u}, u, \bar{v}, v$</td>
<td>$\frac{i(q + m)}{2m^2}$</td>
</tr>
<tr>
<td>$\text{QCD}$</td>
<td>$\bar{u}, u, \bar{c}, c, \bar{a}, a$</td>
<td>$\frac{i(q + m)}{2m^2}$</td>
</tr>
</tbody>
</table>

Okay, so what are $c, \alpha, \lambda, f, g, a, s$?

$C$: There are 3 colors so "charge space" is 3D w/ $\mathbb{C}_0 = (\frac{3}{6}) = \text{red}, \mathbb{C}_2 = (\frac{6}{6}) = \text{blue}, \mathbb{C}_3 = (\frac{3}{6}) = \text{green} basis in $H^3$ space of color states.

An arbitrary state can have $c = \alpha i c$, complex coefficients, hence $c^* = c^{\dagger}$

$a$: There are 8 gluons w/ $a_1 = (\frac{0}{3})$, $a_2 = (\frac{1}{3})$, $a_8 = (\frac{8}{3})$ basis in $H^8$ space of gluon states.

Recall $C = \gamma^\mu C^\nu$, namely, 3x3 = 9 gluons, but the theory only includes 8 transformed into each other by $SU(3)$ (which has 8 generators!)

The "singlet" gluon $\frac{1}{2} (\gamma^\mu + \gamma^5)$ does not exist (would look a lot like $\gamma$ and we haven't seen it!)

Of course we haven't seen the other gluons, but for these we can use the confinement argument.

If the right anti-chiral, then we would say QCD is $SU(3)$ (instead of $SU(8)$).

$\lambda$: For QCD the $\lambda_{ij}^a$ matrices linked "space" to "space-time", i.e. 4 4x4 matrices.

The $\lambda_{ij}^a$ matrices of QCD link $H^3$ of "color space" to $H^8$ of "gluon space", i.e. 8 3x3 matrices ($\lambda_{ij}^a$).

Just like w/ $\gamma$, we will leave off color-space labels and write $\lambda_{ij}$ (and $\bar{c}$ instead of $c^*$).

For $[\lambda_{ij}; \lambda_{kl}^a] = \epsilon_{ijkl} \lambda^a$, structure constants of $SU(3)$ Lie Algebra.

On key technical requirement that exists is to diagram involving internal legs. In this way we have to be very careful not to count gauge equivalent (physically identical) configuration more than once.

To get the boundary right what being gauge invariant, we introduce Fe higgs - Polyakov path which and additional unphysical fields whose sole purpose is to cancel the unphysical gauge equivalent fluctuations of the physical fields.