Brief review of the BHIP

Information $\uparrow$ BH M $\Rightarrow$ BH $\downarrow$ horizon

Maybe okay since information hidden behind horizon

 Thermal Hawking radiation $\Rightarrow T = h/m$

Uh-oh, no more horizon!

If information lost: Non-unitary evolution which means quantum anything gets broken.
If Hawking radiation is wrong: Something off about effective field theory, equivalence principle, etc.

Partial resolution via AdS/CFT $\Rightarrow$ Unitarity must be preserved!

almost anything non-gravity $\Rightarrow$ gravity

But how? While this, and his own similar argument, led Hawking to concede the bet...
the details were still unclear.

So folks kept thinking.
Since these ideas have around entropy and information, let’s sharpen these notions a bit.

**Heuristically:** Entropy is a measure of what we can know, but choose not to know about a system. Entropy is a physically meaningful (and insightful) quantity because the process of gaining knowledge is achieved through physical investigation, i.e., detection and we know that this has physical consequences.

**Classical Thermodynamic Entropy:** For macroscopic systems \( \text{d}E = TdS + \mu dJ + \Phi dQ \) (under reversible changes) is an entropy, \( dS_{\text{sys}} \) is also change in entropy from \( dQ_{\text{sys}} \) not transferred.

\[ E, T, S, \mu, J, \Phi, Q \] are macro state variables

**Statistical Entropy:** Given a coarse-grained description (in terms of average quantities only, for example) there are usually many fine-grained resolutions compatible. If the \( i \)-th compatible resolution (microstate) has probability \( \tilde{p}_i \) to be the true state of the system then \( S_\text{sys} = - \sum_i \tilde{p}_i \ln \tilde{p}_i \). Note for \( \tilde{p}_i = 1 \), \( \tilde{p}_i = 0 \) \( S_\text{sys} = 0 \).

The language we use is that for a coarse-grained description, this is an ensemble of resolutions such that \( \langle \text{what we know} \rangle = \sum \tilde{p}_i \) with resolution \( \tilde{p}_i \).

\[ \text{Microcanonical: } \text{Fix } E + N = \text{equal prob. } \tilde{p}_i = \frac{1}{N} \Rightarrow S_\text{sys} = k \! \ln \! N \text{ or } k \! \ln \! N \text{ for } k = 1 \Rightarrow S_\text{sys} = 0 \]

\[ \text{Canonical: } \text{Fix } T + N \Rightarrow \tilde{p}_i = \frac{1}{N} e^{-E_i/kT} \text{ and } Z = e^{-E_0/kT} \Rightarrow S_\text{sys} = k \! \ln \! Z + k \! \ln \! N \text{ for } T \neq 0 \text{ (only } E_0 \text{ contributes} ) \Rightarrow S_\text{sys} = 0 \]

In all cases \( S \) is extensive, i.e., \( S(A \cup B) = S(A) + S(B) \) (volume scaling).
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