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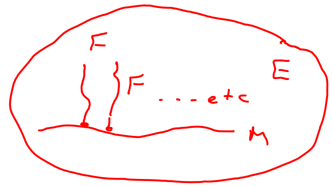
Fiber bundles



Consider 3 differentiable manifolds E, M, and F. Roughly

E ↑ bundle, M ↑ base, F ↑ fiber

- We have a projection $\pi : E \to M$
- We have a Lie group G (the structure group) that acts from the left on F.
- We have an open cover $\{U_i\}$ of the base w/ a diffeomorphism $\phi_i : U_i \times F \to \pi^{-1}(U_i)$

  $\{U_i\}$: set of open and overlapping subregions that in total cover M, i.e. $\bigcup_i U_i = M$

  s.t. $\pi \circ \phi_i (p, f) = p \in M$
- On every non-empty overlap of two subregions $U_i \cap U_j$ we require G-valued transition functions $t_{ij} = \phi_i \circ \phi_j^{-1}$ s.t. $\phi_i = t_{ij} \phi_j$

If we can cover the base w/ one U_i, or if all transition functions can be taken to be trivial, i.e. $t_{ij} = I_{ij} \in G$, then the total bundle is trivial, i.e. $E = M \times F$.

More interesting things arise when this is not the case.

For example take $M = S^1$ and F to be a line segment, i.e. $z \in [-1, 1]$

If we consider $E = S^1 \times [-1, 1]$ then we have a cylinder 

However if we break S^1 into two regions s.t. on one overlap we map $z \to -z$, then we have a mobius strip.

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More practical applications include:

- If F is a vector space then we have a vector bundle
- If F is the tangent space to M, then we are dealing w/ the tangent bundle (important for GR)
- If $F = G$, then we are working w/ a principal bundle (important for gauge theory)

$$C(E) = \det\left(1 + \frac{i}{2\pi} F\right)$$

$F = dA + A \wedge A$ (the curvature 2-form of the bundle)

A: the one-form connection

If E is the tangent bundle then