What does SUSY get us?

As discussed before that (singly gravitino), the SM couplings almost cross at a common value under renormalization. Adding SUSY makes them meet at the same point which strongly hints at unification of $SU(3) \times SU(2) \times U(1)$. 

Moreover, if we consider the renormalization of the Higgs mass (the only fundamental mass in the SM), it should get corrections up to some unification scale, but this would make it too large (it is at the EW scale). Fortunately, adding SUSY has the effect that SM contributions to the running of the Higgs mass are cancelled by contributions from the superpartners. SUSY solves the hierarchy problem.

What about SUSYRA? If we add non-perturbative QG to the SM, we encounter divergences which cannot be renormalized (the theory is formally scale). Adding SUSY does provide some cancellations, but ours still abound.
More is better!

Our discussion so far has been in the context of a single SUSY generator, i.e. N=1 SUSY. But could we add more? Hell yes! And the resulting physics is amazing.

To start, consider N=2. This is literally 2 independent copies of much of what we have talked about. But things get even interesting. Call the SUSY transformations $Q_1$ and $Q_2$. We now have:

- **Hypermultiplets**: $(4, 2\times 0; \bar{\psi}^-)$
- **Vector multiplets**: $(4, 4\times 0; \phi)$
- **Gravitino multiplets**: $(9, 2\times 0; \psi)$

What do we get?

1. We actually “unify” gravity as a normal gauge force since $g_4$ and $\phi$ appear together.
2. The QG divergences actually cancel... but only to one-loop order!

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Even more gooder!

Obey so how big can you go?

a) The Witten-Rabinovitch theorem rules out particles with \( \text{spin} \geq 2 \) in a theory of ...

b) We only want one spin-2 particle, i.e. the graviton.

Consider the \( \mathcal{N}=8 \) gravity multiplet: \( \left( \frac{2}{2} \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} \right) \)

\( (8) \) for \( n \) steps away from 2

If we tried \( \mathcal{N}>8 \), we would invariably end up with something with \( \text{spin} \geq 2 \).

\( \Rightarrow \) \( \mathcal{N}=8 \) SUSY is (11) is the largest possible extension!

What do we get?

1. We "unify" the whole embldde! Everything (and more) appears in the gravity supermultiplet and nothing else is allowed.

2. We get cancellation of \( 5 \text{DRA} 0\)s \( \ldots \) until seventh order!

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Okay, so SUSY and SU(3)GUT are the end all be all to everything.

However, they are great candidates for the low-energy limit of a more fundamental theory which is not problematic... perhaps string theory?

As low-energy limits, the divergences are okay since we know that momentum integrals need to be cut-off $\Gamma$ and replaced by the correct theory above $\Lambda$.

Better still, if we consider the SUSY/SU(3)GUT story in higher dimensions we find an interesting result.
Familiar facts:

$\text{SU}(3)$ → $\text{SU}(2)$

3-comp. $\psi$ (complex) -> comp. spinors

$\text{RAH}$

1+3D $\text{SO}(1,3) \sim \text{SU}(2) \times \text{SU}(2)$

$\psi$ (complex) -> comp. spinors

In general $D$:

Solve $\xi [\gamma^\mu, \gamma^\nu] = 2 \gamma^\rho$ where $\gamma^\mu = \begin{pmatrix} \gamma_0 & \gamma^1 & \cdots & \gamma^D \end{pmatrix}$

for $\gamma^\mu$ which then act

naturally on spinors through

$\gamma_{[\alpha} \gamma_{\beta]} = \delta^\alpha_\beta$ spinor rep. of Lorentz transformation

and by Lorentz algebra

So what does that look like?

In even $D$, i.e. $D = 2k + 2$ we group

$\gamma^a \equiv \frac{1}{k} (\pm \gamma^0 + \gamma_i)$

$\gamma^a_\pm \equiv \frac{1}{k} (\pm \gamma^0 \pm \gamma_{a+1})

=a, \cdots, k$

which satisfy

$\{\gamma^a, \gamma^b\} = \delta^a_b$,

$\{\gamma^a, \gamma^b\} = \{\gamma^a, \gamma^b\} = 0 \Leftrightarrow (\gamma^a)^2 = (\gamma^a)^2 = 0$

Then we can define a "lowest" spinor state such that $\gamma^a \psi = 0$ for all $a$

Then adding in this we get each $\gamma^a$ at most once (since $(\gamma^a)^2 = 0$ we generate $\psi^{(3)}$

where $s = (s_0, s_1, t_1, \cdots, t_k)$ and $s = 0 = (-t, -t, -t, \cdots, -t)$ then each $\gamma^a$ flips one of the $t_i$'s to $-t_i$.

Thus give us $2^k$ different spin states
This yields the $2^{k+1}$ Dirac representation in even D:

$$D = \begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & 2 \end{bmatrix} \quad \text{etc.}$$

$$\begin{bmatrix} 2 & 4 & 8 & 16 \end{bmatrix}$$

This is "reducible" since $\mathbb{S}_{\mathbb{S}^0}$ is quadratic in $\Gamma^8 \Gamma^0$, the 5-states are even, and

\begin{align*}
\text{as of } i & \text{'s do not mix under Lorentz \text{ transformation.}\\
\text{So we can split them into two non-trivial (reducible) sets according to their eigenvalue under} \\
\Gamma' = i \Gamma^0 \Gamma^8 \Gamma^1 \Gamma^0 \Gamma^1 \\
(\text{also called } \Gamma^5 \text{ in \cite{40}})
\end{align*}

The $2^{k+1}$ dimensional Dirac rep. splits into $2 \times 2 (\mathbb{S}_{\mathbb{S}^0}) \text{ spin. Weyl reps.}$

If D is odd, we simply define everything as above, but now use $\Gamma^1 = \Gamma$ will $\Gamma'$ as defined above. In this case the $2^{k+1}$ dim. Dirac rep. is reducible (since $\mathbb{S}_{\mathbb{S}^0}$, $\Gamma^3 = 0$).

Additionally, in some cases we can impose a reality (Hajravan) condition on the Dirac spinors which cuts their d.o.f. in half, and in a few cases we can impose both Hajravan and Weyl conditions leaving half of the Dirac components.

Recall for $N=1$ in $D=4$ we effectively got one useful transformation on massless states (even though in D=4 spinors have 4 components). In general the # of transformations on massless states is equal to the # of spinor components.

Let's consider the smallest spinors in various dimensions:

<table>
<thead>
<tr>
<th>D</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td># comp</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>32</td>
<td>64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(\uparrow\) notice anything special here?