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What does SUSY get us?

We discussed before that (ignoring gravity) the SM couplings <u>almost</u> cross at a common value under renormalization. Adding SUSY makes them meet at the same point which strongly hints at unification of $SU(3) \times SU(2)_L \times U(1)_Y$.

Moreover, if we consider the renormalization of the Higgs mass (the <u>only</u> fundamental mass in the SM) it should get corrections up to some unification scale, but this would make it too large (it is at the EW scale). Fortunately, adding SUSY has the effect that SM contributions to the running of the Higgs mass are cancelled by contributions from the superpartners. SUSY solves the heirarchy problem!

What about SUGRA? If we add naive perturbative QG to the SM, we encounter divergences which cannot be renormalized (the theory is fatally sick).
Adding SUSY does provide some cancellations, but ∞-s still abound.

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More is gooder!

Our discussion so far has been in the context of a single SUSY generator, i.e. $N=1$ SUSY.
But could we add more? Hell yes! And the resulting physics is amazing.

To start, consider $N=2$. This is literally 2 independent copies of much of what we have talked about. But things get more interesting. Call the SUSY transformations Q_1 and Q_2.
We now have:

Hypermultiplet: $(\psi, 2\times\phi, \text{"}-\psi\text{"})$ — The $-$ sign here is an indicator of $\underbrace{\text{helicity opposite that of } \psi}_{\text{a useful categorization of massless states}}$

Vector multiplet: $(V, 2\times\lambda, \phi)$

Gravity multiplet: $(g_{\mu\nu}, 2\times\psi_\mu^\alpha, V)$

What do we get?

1. We actually "unify" gravity w/ a normal gauge force since $g_{\mu\nu}$ and V appear together.
2. The QG divergences actually cancel ... but only to one-loop order!

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Even more is gooderer!

Okay so .. how big can you go?

a) The Weinberg - Witten theorem rules out particles w/ spin > 2 in a theory w/ ...
b) We only want one spin-2 particle, i.e. the graviton.

Consider the N= 8 gravity multiplet: $(2, \underbrace{\tfrac{3}{2}}_{8}, \underbrace{1}_{28}, \underbrace{\tfrac{1}{2}}_{56}, \underbrace{0}_{70}, \underbrace{-\tfrac{1}{2}}_{56}, \underbrace{-1}_{28}, \underbrace{-\tfrac{3}{2}}_{8}, -2)$

$\binom{8}{n}$ for n steps away from 2

If we tried N > 8, we would invariably end up w/ something w/ spin > 2.

$\Rightarrow$ N= 8 SUSY in 4D is the largest possible extension!

What do we get?
1. We "unify" the whole enchilada! Everything (and more) appears in the gravity supermultiplet and nothing else is allowed.
2. We get cancellation of SUGRA ∞-s ... until seventh order!

Okay, so SUSY and SUGRA are the end all fix to everything.

However, they are great candidates for the low-energy limit of a more fundamental theory which is <u>not</u> problematic ... perhaps string theory?

As low-energy limits, the divergences are okay since we know that momentum integrals $\int_0^\infty$ need to be cut-off $\int_0^\Lambda$ and replaced w/ the correct theory above Λ.

Better still, if we consider the SUSY/SUGRA story in higher dimensions we find an interesting result.

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Familiar facts:

NRQM

3D $SO(3) \sim SU(2)$

3-comp vectors 2 (complex)-comp. spinors

RQM

1+3D $SO(1,3) \sim SU(2)\times SU(2)$

4-comp. vectors 4 (complex)-comp. spinors

In general D: Solve $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$ where $\eta^{\mu\nu} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & \ddots \end{pmatrix}$
for Γ's which then act
naturally on spinors through
$e^{-\frac{i}{4}[\Gamma^\mu,\Gamma^\nu]}\psi$ w/ $-\frac{i}{4}[\Gamma^\mu, \Gamma^\nu] = \underbrace{S^{\mu\nu}}_{\text{satisfy Lorentz algebra}}$ spinor rep. of Lorentz transformation

So what does this look like?

In even D, i.e. $D = 2k+2$ we group $\left.\begin{array}{l}\Gamma^{0\pm} = \frac{1}{2}(\pm\Gamma^0 + \Gamma^1) \\ \Gamma^{a\pm} = \frac{1}{2}(\Gamma^{2a} \pm i\Gamma^{2a+1}) \quad a = 1,\dots,k\end{array}\right\}$ like raising and lowering operators!

which satisfy $\{\Gamma^{a+}, \Gamma^{b-}\} = \delta^{ab}$
$\{\Gamma^{a+}, \Gamma^{b+}\} = \{\Gamma^{a-}, \Gamma^{b-}\} = 0 \Rightarrow (\Gamma^{a+})^2 = (\Gamma^{a-})^2 = 0$

Then we can define a "lowest" spinor state such that $\Gamma^{a-}\psi^{(0)} = 0$ for all a

Then acting on this w/ each Γ^{a+} at most once (since $(\Gamma^{a+})^2 = 0$) we generate $\psi^{(s)}$
where $s = (s_0, s_1, s_2, \dots, s_k)$ and $s = 0 = (-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots, -\frac{1}{2})$ then each Γ^{a+} flips one of the $-\frac{1}{2}$'s to $+\frac{1}{2}$.

This gives us 2^{k+1} different spin states

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This yields the 2^{k+1} Dirac representation in even D:

D	2	4	6	8
k	0	1	2	3
2^{k+1}	2	4	8	16

etc.

This is "reducible" since $S^{\mu\nu}$ is quadratic in $\Gamma^\mu\Gamma^\nu$, the s-states w/ even vs. odd #s of $-\frac{1}{2}$'s do not mix under Lorentz trans.
So we can split them into two non-mixing (irreducible) sets according to their eigenvalue under $\Gamma = i^{-k}\Gamma^0\Gamma^1 \cdots \Gamma^{k-1}$ (also called Γ^5 in 4D)

The the 2^{k+1} dimensional Dirac rep. splits into 2 $\frac{1}{2}(2^{k+1})$ dim. Weyl reps.

If D is odd, we simply define everything as above, but now we $\Gamma^d = \Gamma$ w/ Γ as defined above. In this case the 2^{k+1} dim. Dirac rep. is irreducible (since $\{S^{\mu\nu}, \Gamma\} = 0$).

Additionally, in some cases we can impose a reality (Majorana) condition on the Dirac spinors which cuts their d.o.f. in ½, and in a few cases we can impose both Majorana and Weyl conditions leaving $\frac{1}{4}$ of the Dirac components.

Recall for $N=1$ in $D=4$ we effectively got one useful transformation on massless states (even though in $D=4$ spinors have 4 components). In general the # of transformations on massless states is equal to the # spinor comp. /4

Well let's consider the smallest spinors in various dimensions:

d	2	3	4	5	6	7	8	9	10	11	12
# comp	1	2	4	8	8	16	16	16	16	32	64

↑ notice anything special here?