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A status check: 4 observed forces $\underbrace{\text{Electromagnetism, Strong, Weak}}_{\substack{\text{The Standard Model} \\ \text{(a QFT)}}}$ and $\underbrace{\text{Gravity}}_{\substack{\text{General Relativity} \\ \text{(a CFT)}}}$

The programs of unification have different ambitions.

One of the earliest unifications was: $\underbrace{\text{Electricity \& Magnetism}}_{\text{make relativistic}}$

$\Downarrow$

Electromagnetism

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Grand Unified Theories

These are a bit more modest than the name sounds. First of all everything starts relativistic, so nothing new there.

To begin: SM $\overbrace{SU(3)}^{QCD} \times \underbrace{\overbrace{SU(2)_L \times U(1)_Y}^{\text{Electroweak}}}$ Gauge field theory

$\Downarrow$ SSB / Higgs

$SU(3) \times U(1)_{EM}$ (Room Temperature SM)

All of the matter in the SM must fall into representations (reps) of $SU(3) \times SU(2)_L \times U(1)_Y$

For example:

quarks come in 3 color varieties (r, b, g) so are always in the 3 of $SU(3)$, i.e. $(3\times3)\begin{pmatrix} r \\ b \\ g \end{pmatrix}$

leptons do not carry color so are in the 1 of $SU(3)$, i.e. $I_{1\times1}\ (\ell)$

$\begin{pmatrix} u \\ d \end{pmatrix}_L$ quarks and $\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L$ are doublets (the 2) of $SU(2)_L$

etc.

The unification of EM and the Weak interactions can seem mysterious from the bottom up since they seem to play such vastly different roles (nuclear vs. atomic), but from the top down, the differences actually get an explanation ($\underbrace{\text{massive}}_{\substack{\text{short range} \\ \text{w/} \\ \text{larger coupling}}}$ vs. $\underbrace{\text{massless}}_{\substack{\text{long range} \\ \text{w/} \\ \text{smaller coupling}}}$ mediators, etc.).

How do we know EW unification is correct? SSB generally gives rise to mediator mass terms at the symmetry breaking energy scale. We had already "seen" the massive weak bosons prior to GSW putting the theoretical story together.

Important lesson to remember!

Okay, but what do we mean by "unify"? Doesn't $SU(2)_L \times U(1)_Y$ still contain 2 different symmetries?

It is true that $SU(2)_L \times U(1)_Y$ has 2 independent symmetries, but we cannot associate one of them w/ EM and the other w/ W.

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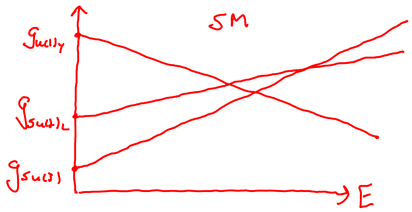
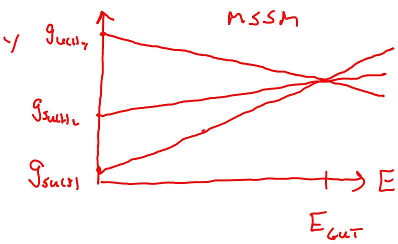
"True" unification of gauge forces would be in terms of a single group w/ a single coupling!

For this to even be a possibility we need $g_{SU(3)} = g_{SU(2)_L} = g_{U(1)_Y} = g_{GUT}$, but we observe $g_S > g_{SU(2)_L} > g_{U(1)_Y}$

However these "run"



But add a little SUSY



← This is one of the main reasons supersymmetry is expected to be a part of fundamental physics.

SU(N) has N^2-1 generators

But what group? Well it has to be big enough to "fit" (actually lin. comb. of these) $\underbrace{SU(3)}_{8\text{ gen.}} \times \underbrace{SU(2)_L}_{3\text{ gen.}} \times \underbrace{U(1)_Y}_{1\text{ gen.}}$ = 12 gen.

The gauge bosons of the SM (8 gluons, $W^\pm$, Z^0 and γ) correspond to the 12 gen. above, i.e. transform in adjoint.

It also has to fit the representations of quarks and leptons, i.e.

Note: anything w/ $M \neq 0$ requires L+R comp.

$$\underbrace{\left[\ \underset{\text{colors}}{3\times}\begin{pmatrix}u\\d\end{pmatrix}_L,\ \underset{\text{colors}}{3\times u_R},\ 3\times d_R,\ 1\times\begin{pmatrix}e\\ \nu_e\end{pmatrix}_L,\ 1\times e_R\ \right]}_{15\ \text{matter fields}} \times 3 \text{ generations} \quad \begin{pmatrix}u\\d\end{pmatrix}\begin{pmatrix}c\\s\end{pmatrix}\begin{pmatrix}t\\b\end{pmatrix} \quad \begin{pmatrix}e\\ \nu_e\end{pmatrix}\begin{pmatrix}\mu\\ \nu_\mu\end{pmatrix}\begin{pmatrix}\tau\\ \nu_\tau\end{pmatrix}$$

The smallest single group that does the job is SU(5) which has 24 generators of which

$$8 \text{ are } \underbrace{\begin{pmatrix}M^{3\times3} & 0\\ 0 & 0\end{pmatrix}}_{SU(3)},\quad \underline{\text{commutes w/}} \quad 3 \text{ are } \underbrace{\begin{pmatrix}0 & 0\\ 0 & M^{2\times2}\end{pmatrix}}_{SU(2)_L} \quad \underline{\text{commutes w/}} \quad \text{and } 1 \text{ is } \underbrace{\begin{pmatrix}-\tfrac{1}{3} &&&& \\ & -\tfrac{1}{3} &&& \\ && -\tfrac{1}{3} && \\ &&& \tfrac{1}{2} & \\ &&&& \tfrac{1}{2}\end{pmatrix}}_{U(1)_Y} + 12 \text{ more}$$

And the 15 matter fields fit into the 5^* $\underbrace{\text{(conjugate vector)}}_{1\times5}$ and 10 $\underbrace{\text{(anti-symm. tensor)}}_{\text{anti-symm } 5\times5}$ lowest dim. reps. of SU(5)!

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Okay, so besides being "pretty" or "simpler", does this really get us anything?

- A single simple GUT gauge group, e.g. $SU(5)$, would explain why electric charge is quantized (essentially there is no abelian factor like $U(1)_Y$, and the nontrivial commutation relations between all generators produce quantization of their eigenvalues)
- We also learn why the charge of the proton is exactly equal and opposite the charge of the electron (essentially since the quarks and electron are in a single rep. of $SU(5)$, the trace of the electric charge operator over the rep. must vanish which ultimately implies $3Q_d = Q_{e^-}$, from which one can get to $Q_p = 2Q_u + Q_d = -Q_{e^-}$.)

Can we do better? It turns out that one get an even more unified version of the matter fields (all for each generation in a single rep.) if one is willing to go a little bigger.

$SO(10)$: $\frac{1}{2}10(10-1) = 45$ generators

This may seem unnecessarily big, but remember that we are embedding unitary (complex) groups into this real group, and complex numbers carry more information than their real counterpart, e.g. $SU(2) \sim SO(3)$

How do things fit? For starters, since we know $SU(5)$ works well for the gauge bosons so let's start w/ the generators:

$SO(10) \supset SU(5)$

$$\underset{\substack{\text{adjoint}\\ \text{of } SO(10)}}{45} \to \underset{\substack{\text{adjoint}\\ \text{of } SU(5)}}{24} \oplus \underset{\text{singlet}}{1} \oplus \underset{\text{anti.}}{10} \oplus \underset{\substack{\text{conj.}\\ \text{anti.}}}{10^*}$$

What about matter? Well we have 15 things to shove somewhere. The fundamental (or vector) of $SO(10)$ is a $\underbrace{10}_{\text{too small}}$. The anti. tensor is $\frac{1}{2}(10\times 10 - 10) = \underbrace{45}_{\text{too big}}$ (also the adjoint)

But what about the spinor? For $SO(10)$ an irreducible spinor has $2^{\frac{10}{2}-1} = 16$ comp.

$SO(4)$ $2^{\frac{4}{2}-1} = 2$ for a left or right Weyl spinor

16 is close, but it would require one more field... ν_R!

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In any of these GUT schemes, there are several very important problems to be addressed:

- Remember those "extra" generators, well they are new gauge bosons w/ mass $\sim M_{GUT} = E_{GUT}$ (suppressed below E_{GUT}). Since quarks and leptons are in the same rep. of the interactions, the (heavier) quarks can be transformed into (lighter) leptons which means the proton can decay. But we know $\tau_p > 10^{31}$ years, so this means M_{GUT} must be huge. Well in fact the unification of couplings occurs around $E_{GUT} \sim 10^{16}$ GeV which works!

- Since $M_{Higgs} \sim M_{EW} \sim 10^2$ GeV $\ll M_{GUT}$, one must argue as to why the Higgs mass does not receive radiative corrections (which would drive its value up to M_{GUT}). The rest of the SM particles are massless above M_{EW}! The good news is that this "hierarchy problem" is addressed by SUSY which brings in superpartners whose loop corrections cancel those of ordinary matter.

Enough is enough! Time to move on...