A status check: 4 abeved forces, Electricity, Strong, Weak, and Gravity. The Standard Model (a QFT) and General Relativity (a CFT).

The programs of unification have different ambitions.

One of the earliest unifications was: Electricity + Magnetism make relativistic ∇"Electromagnetism."
Grand Unified Theories

These are a bit more radical than the same sounds. First of all, everything starts relativistic, so nothing new there.

\[
\begin{align*}
\text{GUT} & \quad \text{SU}(3) \times SU(2)_L \times U(1)_Y \\
\text{Electroweak} & \quad SU(2)_L \times U(1)_Y \\
\text{Higgs} & \quad \text{SU(3)} \times \text{U(1)}_A
\end{align*}
\]

To begin: \[ \text{SM} \quad \text{SU}(3) \times SU(2)_L \times U(1)_Y \quad \text{Gauge Field theory} \]

All of the matter in the SM must fall into representations (reps) of \[ \text{SU}(3) \times SU(2)_L \times U(1)_Y \]

For example:

quarks come in 3 color varieties \((1, 3, 0)\) so are always in the \(3\) of \(\text{SU}(3)\), i.e. \((3 \times 3) \left( \frac{1}{3} \right) \)

leptons do not carry color so are in the \(1\) of \(\text{SU}(2)_L\), i.e. \((3 \times 1) \left( \frac{1}{2} \right) \)

\((u, d)\) quarks and \((e, \nu)\) are doublets (the \(2\) of \(\text{SU}(2)_L\), etc.

The unification of EM and the weak interactions can seem mysterious from the bottom up issue they seem to play such vastly different roles (nuclear vs. atomic), but from the top down, the differences actually get an explanation (long vs. short range, larger coupling vs. smaller coupling).

How do we know EM unification is correct? / SSB generally gives rise to mediator mass terms at the symmetry breaking energy scale. We had already

Inverted lepton to remember! \{ “seen” the massive weak bosons prior to GSW putting the theoretical story together.

Okay, but what do we mean by “verify”? Does \[ \text{SU}(3) \times SU(2)_L \times U(1)_Y \] still contain 2 different symmetries?

\[ \text{It is true that SU}(3) \times SU(2)_L \times U(1)_Y \text{ has } 2 \text{ independent symmetries, but we cannot associate one of them with EM and the other with W.} \]
"True" unification of gauge forces would be in terms of a single group with a single coupling!

For this to even be a possibility we need $g_s = g = g = g_{SW}$, but we observe $g > g_{SW} > g_{SW}$.

However these "true"

But add a little SUSY

But what group? Well it has to be big enough to "fit" $SU(3) \times SU(2) \times U(1)$.

The gauge bosons of the SM (8 gluons, $W^\pm$, $Z^0$, and $Y$) correspond to the 12 gen of above, i.e. transform in adjoint.

We also have to fit the representations of quarks and leptons, i.e.

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The smallest single group that does the job is $SU(5)$ which has 27 generators of which

8 are $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, 3 are $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and 1 is $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (gauge singlet).

$SU(5)$ contains $SU(3)$ $SU(2)$ and $U(1)$.

And the 15 matter fields fit into the $5^* (24, singlet)$ and $10 (5-3-3, triplet)$ lowest dim. reps. of SU(5).
Okay, so besides being "pretty" or "simple", does this really get us anything?

- A single simple GUT gauge group, e.g. SU(5), would explain why electric charge is quantized (essentially there is no abelian factor like U(1), and the maximal commutation relations between all generators produce quantization of their eigenvalues).
- We also learn why the charge of the proton is exactly equal and opposite the charge of the electron (essentially since the quarks and electron are in a single rep. of SU(5), the trace of the electric charge operator over the rep. must vanish which ultimately implies 
\[ 3Q_d = Q_e \] , from which one can get to \[ Q_p = 2Q_u + Q_d = -Q_e. \])

Can we do better? It turns out that one can get an even more unified version of the matter fields (all for each generation in a single rep.) if one is willing to go a little bigger.

\[ SO(10) : \frac{1}{2} (10 + \overline{10}) = 15 \] generators.

This may seem unnecessarily big, but remember that we are embedding unitary (complex) groups into this real group, and complex numbers carry more information than their real counterpart, e.g. \[ SU(2) \to SO(3) \]

How do things fit? For starters, since we know SU(5) works well for the gauge bosons, so let's start with the generators:

\[ SO(10) \supset SU(5) \]

\[ 10 \rightarrow \begin{array}{c} 1 \oplus 10 \oplus 10^* \end{array} \]

\[ SU(5) \text{ adj.} \quad SO(10) \text{ adj.} \quad SU(5) \text{ sing.} \quad SO(10) \text{ sing.} \]

What about matter? Well we have 15 things to chose somewhere. The fundamental (or vector) of \[ SO(10) : a 10 \). The anti-tensor is \[ \frac{1}{2} (10 \times 10 - 10) = 45 \] (also the adjoint).

Can the small by big?

But what about the spinor? For \[ SO(10) \text{ an irreducible spinor has } 2^{16} = 64 \text{ comp.} \]

\[ SO(4) \times SU(2), \text{ for a left or right weak spinor} \]

\[ 16 \text{ is clear, but it would require one more field...} \]

GUTs, TOEs and Strings Page 4
In any of these GUT schemes, there are several very important problems to be addressed:

• Remember those “extra” generators, well they are new gauge bosons of mass $M_{\text{GUT}} \approx E_{\text{GUT}}$ (supposed below $E_{\text{W}}$). Since quarks and leptons are in the same reg. of the interactions, the (heavier) quarks can be transformed into (lighter) leptons which means the proton can decay. But we know $T_{\pi} > 10^{33}$ years, so this means $M_{\text{GUT}}$ must be huge. Well in fact the unification of couplings occurs around $E_{\text{GUT}} \approx 10^{16}$ GeV which works!

• Since $M_{\text{Higgs}} \approx M_{\text{W}} < M_{\text{GUT}}$, one must argue as to why the Higgs mass does not receive radiative corrections (which would drive its value up to $M_{\text{GUT}}$). The rest of the SM particles are heavier above $M_{\text{EW}}$. The good news is that this “hierarchy problem” is addressed by SUSY which brings in superpartners whose loop corrections cancel those of ordinary matter.

Enough is enough! Time to move on...