Monday, March 18, 2019
5:28 PM

Review I:

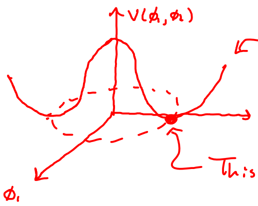
For closed and oriented strings, finiteness at 1-loop requires:

a) Modular invariance of partition function $\quad \tau = \tau_1 + i\tau_2 : \tau \to \tau + 1 , \tau \to -\frac{1}{\tau}$

b) contributions from all closed string states, i.e. all solutions to the 2D field theory w/ prescribed boundary conditions

Review II:

A Lagrangian or action may exhibit a symmetry, but a solution to the e.o.m. need not share that symmetry.



$V(\phi_1,\phi_2)$ is symmetric under rotations in ϕ_1-ϕ_2 plane

This solution $\phi = (\phi_1, \phi_2) = (0, \phi_0)$ is not invariant

Review III:

Requiring spacetime fermions and no tachyon $\Rightarrow$ 5 superstring theories in $\mathbb{M}^{10}$: $\mathcal{N} = 32$ IIA, IIB

$\mathcal{N} = 16$ Het (E8×E8), Het (SO(32))

Type I (w/ open strings)

We are going to look at some connections between these from a powerful tool called "twisting" which gives a new theory on $\mathbb{M}^{10}$.
We will then look at other examples of twisting which can change the underlying geometry and/or require brand new d.o.f.

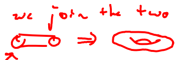
Monday, March 18, 2019
5:38 PM

Twisting

Start w/ any consistent string theory based on oriented closed strings w/ a MI partition function.

Consulting the worldsheet action we may find some discrete symmetry transformation g which leaves S invariant. We may consider projecting the spectrum of the theory by $P = \frac{1}{2}(1+g)$ where $P^2 = P$ if $g^2 = I$, i.e. $g \in \mathbb{Z}_2$.
Projecting the partition function we form:

$$Z_{T^2} \to Z_{T^2}[P] = Z_{T^2}[\tfrac{1}{2}] + Z_{T^2}[\tfrac{1}{2}g] = \tfrac{1}{2} Z_{T^2} + \tfrac{1}{2} \underbrace{Z_{T^2}[g]}$$

this means when we join the two ends of the cylinder to make  we "twist" or operate on one w/ g.

We know that $\frac{1}{2} Z_{T^2}$ is MI, but in general we do not expect $Z_{T^2}[g]$ to be. One reason for this is that by projecting w/ P, we are throwing away some string states, but we know we need them all for MI.

Thus if $Z_{T^2}[P]$ is not MI, but Z_{T^2} is, then $Z_{T^2}[g]$ can't be.

There are two approaches to fixing this: a) Take $Z_{T^2}[g]$ and keep applying $\tau \to \tau + 1$ and $\tau \to -\frac{1}{\tau}$ until we generate new terms which together with $Z_{T^2}[g]$ form a closed (hence invariant) set.

b) Think about new string states that should be added, do so and then project them as well.

Both approaches get often get to the same result, and indeed route (b) was used to "explain" the new terms required by (a).

Monday, March 18, 2019
5:49 PM

Examples:

IIA twisted by an action on WS fermions gives IIB
Het (SO(32)) twisted by an action on gauge fermions gives Het ($E8 \times E8$)

Perhaps more interestingly consider the symmetry group of an action based on $X^{\mu} X^{\nu} \eta_{\mu\nu}$, i.e. Poincaré in D-dim.