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Supersymmetry (affectionately known as SUSY) is a rather old (~60s) idea that is the most favorable, but still untested, extension of the Standard Model. It is deep w/ a lot of notational complexity and profound mathematical consequences. Unfortunately it is difficult to give it any intuitive meaning for reasons we will see shortly.

As it is best understood, the Standard Model of particle physics has a Lagrangian whose form is based on the following symmetries:

Spacetime: The Poincaré group of $\mathbb{M}^4$ which is $\mathbb{R}^{1,3} \ltimes SO(1,3)$

Internal (or gauge): $\underbrace{SU(3)}_{QCD} \times \underbrace{SU(2)_L \times U(1)_Y}_{EW} \xrightarrow[\text{Higgs}]{} \underbrace{SU(3)}_{QCD} \times \underbrace{U(1)_{EM}}_{EM}$

$\Big\}\ \mathbb{R}^{1,3} \ltimes SO(1,3) \times SU(3) \times SU(2)_L \times U(1)_Y$

All of these symmetries correspond to Lie algebras:

$$\mathbb{R}^{1,3} \ltimes SO(1,3): \begin{cases} [P_\mu, P_\nu] = 0 \\ [M_{\mu\nu}, P_\rho] = i\eta_{\mu\nu}P_\nu - i\eta_{\nu\rho}P_\mu \\ [\underset{\sim}{M_{\mu\nu}}, M_{\rho\sigma}] = i\eta_{\mu\rho}M_{\nu\sigma} - i\eta_{\mu\sigma}M_{\nu\rho} - i\eta_{\nu\rho}M_{\mu\sigma} + i\eta_{\nu\sigma}M_{\mu\rho} \end{cases}$$

includes $\{J_m, K_n\}$ the generators of rotations and boosts

SU(3) structure constants

$$SU(3): [g^a, g^b] = i\overset{\frown}{f^{abc}}\, g^c$$
$$SU(2)_L: [g^i, g^j] = i\epsilon^{ijk} g^k$$
$$U(1): [g, g] = 0$$

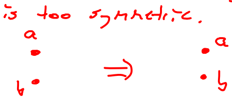
There is already a sense in which some of the internal symmetries are unified in the electroweak sector, and indeed the aim of grand unified theories is to unite the EW with QCD which we talk about next time.

But before this story was even complete, physicists found it useful to try and "combine" internal symmetries with those of spacetime. This did not get very far due to the work of Coleman and Mandula who formulated a "no-go" theorem.

First of all what would it mean to "combine" symmetries? Well at present, if we take any generator from $\mathbb{R}^{1,3} \ltimes SO(1,3)$ and any internal symmetry generator we find $[P_\mu, g] = 0$, $[M_{\mu\nu}, g] = 0$

To combine then means that some g would have to have nontrivial commutators, i.e. $[P_\mu, g] \neq 0$, $[M_{\mu\nu}, g] \neq 0$ which can be turned into the statement the $\underset{\smile}{g}$ would have to transform as a (nontrivial) tensor under $\mathbb{R}^{1,3} \ltimes SO(1,3)$.

But CM showed that such a scenario is too symmetric. The spirit of their argument was in analyzing 2-body scattering



The conservation of energy, momentum and angular momentum constrain the final state so that there is only one unknown (the scattering angle). These of course follow from the symmetries under P_μ and $M_{\mu\nu}$. However if we add an additional spacetime symmetry g, then we get another conservation law which would discretize the allowed values of the scattering angle θ. But the scattering amplitude is an analytic function of θ, so if θ can only take discrete values, then the amplitude itself must be 0. That is, the theory is non-interacting!

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Now the Coleman Mandula theorem rests upon several assumptions. One of which is that the symmetries in question are bosonic, i.e. they have algebras associated w/ commutation relations.

That is, if we quantize a bosonic field, then the creation/annihilation operators satisfy commutation relations which among other things allow multi particle excitations in the same state, e.g. $a_{(b)}a_{(b)}\phi \neq 0$ is consistent w/ $[a_{(b)}, a_{(b)}] = 0$ which is necessary.

However we could consider fermionic symmetries which utilize anticommutators. Again more familiarly one can show that $a_{(f)}a_{(f)}\phi = 0$ is required since $\underbrace{\{a_{(f)}, a_{(f)}\}=0}$ which is the Pauli-Exclusion Principle.

$$a_{(f)}a_{(f)} + a_{(f)}a_{(f)} = 0 \Rightarrow \underbrace{a_{(f)}a_{(f)} = -a_{(f)}a_{(f)}}_{0=0}$$

So in the end, there turns out to be one unique avenue to extending the family of spacetime symmetries, and that is by adding a fermionic spacetime symmetry w/ the full algebra:

(N=1) SUSY:

$$[P_\mu, P_\nu] = 0$$
$$[M_{\mu\nu}, P_\rho] = i\eta_{\mu\rho}P_\nu - i\eta_{\nu\rho}P_\mu$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = i\eta_{\mu\rho}M_{\nu\sigma} - i\eta_{\mu\sigma}M_{\nu\rho} - i\eta_{\nu\rho}M_{\mu\sigma} + i\eta_{\nu\sigma}M_{\mu\rho}$$
$$[Q_\alpha, P_\mu] = 0$$
$$\{Q_\alpha, Q_\beta\} = 0$$
$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}}P_\mu$$

Through the spin-statistics theorem, the fermionic symmetry operators are $\frac{1}{2}$-integer spin. But this has an unusual consequence. We are used to acting w/ bosonic operators, for example a Lorentz transformation of a vector $A^\mu \to A^{\mu'} = \Lambda^{\mu'}{}_\nu A^\nu$ gives us back another vector. This is in a sense due to the fact that the generators themselves are tensors carrying the right index structure to return a vector. In fact it should be obvious that any bosonic operator acting on any tensor will return a tensor. Less obvious is that if we act w/ a bosonic operator on a spinor, we get back another spinor (albeit we have to use the spinor representation of the bosonic operator), e.g. $\psi \to \psi' = \underbrace{S[\Lambda]}_{e^{\frac{i}{4}[\gamma^\mu,\gamma^\nu]}}\psi$.

This is all consistent w/ angular momentum addition, i.e.

$$\text{Int} \oplus \text{Int} = \text{Int}$$
$$\text{Int} \oplus \tfrac{1}{2}\text{Int} = \tfrac{1}{2}\text{Int}$$

However, when we act w/ these new fermionic transformations, the shift the spin by $\frac{1}{2}$, e.g.

$$\tfrac{1}{2}\text{Int} \oplus \text{Int} = \tfrac{1}{2}\text{Int}$$
$$\tfrac{1}{2}\text{Int} \oplus \tfrac{1}{2}\text{Int} = \text{Int}$$

So what we find is that these new transformations take bosons into fermions and vice-versa.

BTW you can also see this w/ the index structure since Q_α carries a spin index, then $Q_\alpha A_\mu = B_{\alpha\mu}$ (spin $\frac{3}{2}$).

The algebra element $\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$ has several crucial consequences.

1) Since the SUSY transformation is tied to spacetime translations, SUSY must act on every field in the theory (since translations do). Unlike for example $U(1)_{EM}$ which does not act on neutrinos since they carry zero electric charge, everything is charged under SUSY.

2) If (like the internal symmetries of the SM) we wanted to take this global transformation and gauge it, i.e. $Q_\alpha \to Q_\alpha(x^\mu)$, then we would be forced to also gauge translations. But a theory of gauged translations is GR! So gauging SUSY necessarily incorporates gravity leading to SUGRA (Supergravity).

If we want to construct an invariant action, then we need "SUSY scalars". Of course we must also be working w/ Lorentz scalars. As a simple example consider:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2\phi^*\phi + i(\hbar c)\bar\psi\gamma^\mu\partial_\mu\psi - (mc^2)\bar\psi\psi$$

If we impose a SUSY transformation which takes the form (in this notation) then one can show that $\mathcal{L}$ is unchanged up to a total divergence (which will not change the resulting e.o.m.).

$$\phi \to \phi + \delta\phi = \phi + 2\bar{Q}\psi$$
$$\psi \to \psi + \delta\psi = \psi - \left(\frac{i}{\hbar c}\right)\gamma^\mu Q\partial_\mu\phi$$

But, this result is contingent on ϕ and ψ having the same mass and all other internal quantum #s.

So in a supersymmetric theory, every particle has a superpartner w/ spin different by 1/2.

This is <u>not</u> observed! But that's okay, because we know that symmetries can be present but hidden (not manifest) if they are spontaneously broken. If this happens at the EW breaking scale, then the superpartners could get (huge) masses from the Higgs mechanism.

so big they are beyond currently accessible energies

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Okay, but is SUSY a good thing?

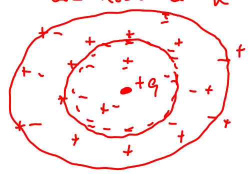
Well first of all the lightest superpartner particles would be stable against decay and so serve as a reasonable candidate for dark matter.

SUSY is actually a requirement of stable string theories.

To understand the other benefits of SUSY we need a 2-minute primer on renormalization.

Analogy, classical charge screening:

Charge inside polar medium:



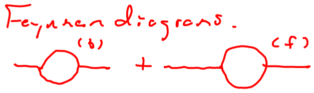
At larger distances we can cut more pairs in half by Gaussian spheres so the internal charge drops w/ distance. The true charge is measured @ $r = 0$.

In QFT the same thing happens from virtual e^+e^- pairs, but w/out any lattice spacing. So in QFT electric charge (and in fact all couplings) "run" w/ changes on energy scale (distances probed).

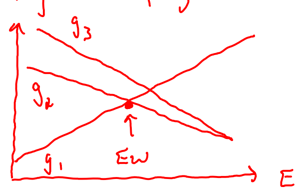
This leads to 2 amazing effects of SUSY:

Recall that the mass of the Higgs is $\frac{\sqrt{2}\hbar}{c}\mu$ where μ is the quadratic Higgs self-coupling. This coupling (like all others) will run, but due to its particular running it will receive corrections that should make its current value immensely larger than observed. The only way to explain this is to argue that above E_{Higgs}, some delicate cancellations are happening.

But w/ SUSY, they do! Coupling corrections come from loops in Feynman diagrams. In SUSY theories these come in pairs and get a relative minus sign in their contributions to amplitudes. So they cancel!



Calling the couplings of EM, W and QCD g_1, g_2, g_3 we find that:



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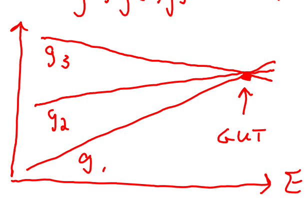

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More on this next time!