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Spinors are weird

I like to motivate these in Particle Physics w/ the following example:

How do I compute/define $\sin\theta$ for a given θ?  then $\sin\theta = \frac{o}{h}$ ← measure ← measure

This definition is rooted in a concrete physical/spatial context.

But does it allow us to evaluate $\sin(i\theta)$? We need a more general definition.

Taylor series: $\sin\theta = \theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 + \cdots$ Then just plug in θ or $i\theta$ to get answer.

So sometimes we need an abstract definition to be able to apply things beyond their more obvious spatially rooted contexts.

Vectors (or more generally tensors) are intimately tied to coordinates.
Vectors have a component for each coordinate used to describe a space or spacetime.
Their components transform under a coordinate change w/ some linear operator, e.g. a rotation or boost matrix. Scalars by definition do not transform. Higher rank tensors are generalizations of vectors.

Suppose $D=3$ + the $\vec{V}$ has 3-components and under a rotation $V \to V' = \underset{\uparrow\, 3\times 3}{R}\, V$

Can we define this more abstractly? $R = e^{i g^a \theta^a}$ where the generators $\underbrace{g^1, g^2, g^3}_{3\times 3 \text{ matrices}}$ induce rotations by $\theta^1, \theta^2, \theta^3$ in the x-y, y-z, z-x planes respectively.
We could define rotations by the set of R's $(SO(3))$ or in terms of the generators, or better still the "algebra" of the generators $[g^a, g^b] = i\epsilon^{abc} g^c$

But using the algebra as a starting point allows us to find other possibilities:
$\sigma^1, \sigma^2, \sigma^3$ are complex 2×2 matrices which satisfy this algebra.
Forming $R = e^{i\sigma^a\theta^a}$ we get a 2×2 complex rotation matrix $(SU(2))$

What do these act on? Complex 2-component objects called spinors!

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Last year we learned that the Coleman-Mandula theorem prevented any possible extension of the spacetime symmetries of QFT, i.e. Poincaré $= \mathbb{R}^4 \ltimes SO(1,3)$ for symmetries w/ generators satisfying a Lie Algebra, i.e. satisfying commutation relations.

To get around it, the trick was to consider new symmetries with generators that satisfy anticommutation relations, i.e. the $\underbrace{(\tfrac{1}{2},0) \text{ and } (0,\tfrac{1}{2})}$ spinor representations of the Lorentz group.

Take the 6 generators of $SO(1,3)$
form 2 sets of complex combinations
which split into $\underbrace{SU(2)}_{(\frac{1}{2},0)} \times \underbrace{SU(2)}_{(0,\frac{1}{2})}$

Now I want you to think about this for a moment. We have certainly encountered fields (and particle states) in the SM which transform in the spinor representation of $SO(1,3)$, e.g. all SM "matter" are spin-$\tfrac{1}{2}$ fermions. But this is saying something new, and woefully more complicated. This is saying that the new transformations themselves are spinors!

Compare: Translations are generated by linear momentum $\Rightarrow$ tensorial } When acting on
Rotations are generated by angular momentum $\Rightarrow$ tensorial } bosons <u>or</u> fermions
Supersymmetry is generated by spinors and can act on bosons or fermions.

The full SUSY algebra is:

$$[P_\mu, P_\nu] = 0$$
$$[M_{\mu\nu}, P_\rho] = i\eta_{\mu\rho} P_\nu - i\eta_{\nu\rho} P_\mu$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = i\eta_{\mu\rho} M_{\nu\sigma} - i\eta_{\mu\sigma} M_{\nu\rho} - i\eta_{\nu\rho} M_{\mu\sigma} + i\eta_{\nu\sigma} M_{\mu\rho}$$
$$[Q_\alpha, P_\mu] = 0$$
$$\{Q_\alpha, Q_\beta\} = 0$$
$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

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So the first thing one has to do is work out the representation theory for this new set of transformations.

Lightning review of rotations in 3D: S_x, S_y, S_z do not commute → S^2 does (Casimir invariant)
So choose eigenvalues of S^2 & S_z to classify.
$S^2 = 0, \frac{1}{2}, 1$, etc. (denotes how they transform)

$\underbrace{S_z = 0}_{\text{scalars}}$ $\underbrace{S_z = \pm\frac{1}{2}}_{\text{spinors}}$ $\underbrace{S_z = 0, \pm 1}_{\text{vectors}}$ (denotes effects of $\mathbb{R}$)

To develop the representation theory of SUSY is way beyond us. But the answer is:

SM matter fermions
scalar superpartners (selectron, sneutrino, squarks, etc.)

Chiral supermultiplets (ψ, ϕ)
Vector supermultiplets (V, λ)

In each case the SUSY transformation takes us from left to right, decreasing the spin by $\frac{1}{2}$.

spinor superpartners (photino, gluinos, winos and zinos)
SM gauge bosons

This actually covers the entire content of the MSSM except the Higgs which is also part of a chiral multiplet but this time (ψ, ϕ)
Higgsino Higgs

You may ask: What about gravity? Well normally any discussion of the SM ignores gravity (set in $\mathbb{M}^4$ flat spacetime).

But, if we get frisky and consider gauging this new transformation, then according to: $\{\underbrace{Q_\alpha, \bar{Q}_{\dot{\beta}}}_{\text{making these local}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu$
means making P_μ local

But a theory of gauged 4-translations is exactly GR!

So gauging SUSY → SUGRA and we then get to add $(\underbrace{g_{\mu\nu}}_{\text{spin } 2}, \underbrace{\psi^\alpha_\mu}_{\text{spin } \frac{3}{2}})$ to the mix.
graviton gravitino

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What does SUSY get us?

We discussed before that (ignoring gravity) the SM couplings <u>almost</u> cross at a common value under renormalization. Adding SUSY makes them meet at the same point which strongly hints at unification of $SU(3) \times SU(2)_L \times U(1)_Y$.

Moreover, if we consider the renormalization of the Higgs mass (the <u>only</u> fundamental mass in the SM) it should get corrections up to some unification scale, but this would make it too large (it is at the EW scale). Fortunately, adding SUSY has the effect that SM contributions to the running of the Higgs mass are cancelled by contributions from the superpartners. SUSY solves the heirarchy problem!

What about SUGRA? If we add naive perturbative QG to the SM, we encounter divergences which cannot be renormalized (the theory is fatally sick).
Adding SUSY does provide some cancellations, but ∞-s still abound.