Spinos are weird.

I like to motivate these in Particle Physics at the following examples:

How do I compute define \( \sin \theta \) for a given \( \theta \)? \( \sin \theta = \frac{\theta}{\theta} \) then \( \sin \theta = \frac{\theta}{\theta} \) measure.

This definition is rooted in a concrete physical/spatial context.

But does it allow us to evaluate \( \sin(i\theta) \)? We need a more general definition.

Taylor series: \( \sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots \). Then just plug in \( \theta \) or \( i\theta \) to get answer.

So sometimes we need an abstract definition to be able to apply things beyond their more obvious spatially rooted contexts.

Vectors (or more generally tensors) are intimately tied to coordinates.

Vectors have a component for each coordinate used to describe a space or spacetime.

Their components transform under a coordinate change and some linear operators, e.g. a rotation or boost matrix. Scalars by definition do not transform. Higher rank tensors are generalizations of vectors.

Suppose \( \text{D} = 3 \) the \( \mathbf{u} \) has 3 components and under a rotation \( U \) \( \mathbf{u} \rightarrow U \mathbf{u} \in \mathbb{R}_3 \times \mathbb{R}_3 \).

Can we define this more abstractly? \( R = e^{i \gamma a} \) where the generators \( \gamma^0, \gamma^1, \gamma^2 \) induce rotations by \( \gamma^0, \gamma^1, \gamma^2 \) in the \( x-y, y-z, z-x \) planes respectively. \( 3 \times 3 \) matrices.

We could define rotations by the set of \( \gamma^a \) \( (\mathbb{R}(3)) \) or in terms of the generators, or better still the "algebra" of the generators \( \{\gamma^a, \gamma^b\} = i \epsilon^{abc} \gamma^c \).

But using the algebra as a starting point allows us to find other possibilities; \( \gamma^a, \gamma^b \) are complex \( 2 \times 2 \) matrices which satisfy this algebra.

Forming \( R = e^{-\theta \gamma^a} \) we get a \( 2 \times 2 \) complex rotation matrix \( (SU(2)) \).

What do these act on? Complex 2-component objects called spinors!
Last year we learned that the Coleman-Mandula theorem prevented any possible extension of the spacetime symmetries of QFT, i.e. Poincaré = IR^4 x SO(1,3) for symmetries with generators satisfying a Lie Algebra, i.e. satisfying commutation relations.

To get around it, the trick was to consider new symmetries with generators that satisfy anti-commutation relations, i.e. the $\left(\frac{1}{2}, 0\right)$ and $(0, \frac{1}{2})$ spinor representations of the Lorentz group.

Take the 6 generators of $SO(1,3)$ form 2 sets of complex combinations which split into $SU(2) \times SU(2)$

\[ \left(\frac{1}{2}, 0\right) \quad (0, \frac{1}{2}) \]

Now I want you to think about this for a moment. We have certainly encountered
fields (and particle states) in the $5\mathbf{h}$ which transform in the spinor representation of $SO(1,3)$, e.g. all $5\mathbf{h}$ “matter” are spinor fermions. But this is saying something new, and wonderfully more complicated. This is saying that the new transformations themselves are spinors!

Compare: Translations are generated by linear momentum $\rightarrow$ tensorial (When acting on a vector, the generators are tensorial.)
Rotations are generated by angular momentum $\rightarrow$ tensorial (When acting on a basis of fermions, supersymmetry is generated by spinors and can act on bases or fermions.

The full SUSY algebra is:

\[ [\gamma^\mu, \gamma^\nu] = 0 \]
\[ [\gamma^\mu, \gamma^\nu] = i \gamma^\mu \gamma^\nu - i \gamma^\nu \gamma^\mu \]
\[ [\gamma^\mu, \gamma^\nu] = i \gamma^\mu \gamma^\nu - i \gamma^\nu \gamma^\mu \]
\[ [Q_\alpha, \gamma^\mu] = 0 \]
\[ \{Q_\alpha, Q_\beta\} = 0 \]
\[ \{Q_\alpha, \bar{Q}_\beta\} = 2 \delta^\alpha_\beta P_\mu \]
So the first thing one has to do is work out the representation theory for this new set of transformations.

Lightning review of rotations in 3D: $S_3, S_2, S_1$ do not commute $\Rightarrow S_2$ does (Casimir invariant)
$S_2$ has eigenvalues of $S_2$, $S_2 \pm 1$, etc. (denotes how they transform)

\[ S_2 = 0 \quad S_2 = 1 \quad S_2 = 2 \quad \text{etc.} \]

\[ \begin{array}{ccc}
\text{scalars} & \text{spins} & \text{vectors} \\
S_2 = 0 & S_2 = 1 & S_2 = 2 \end{array} \]

To develop the representation theory of SUSY is way beyond us. But the answer is:

- $\phi$ = scalar superpartners (scalars, scalars, squarks, etc.)
- $\chi$ = chiral supermultiplets ($\psi$, $\phi$)
- $\psi$ = vector supermultiplets ($\chi$, $\chi$)

In each case, the SUSY transformation takes us from left to right, increasing the spin by 1.

This actually covers the entire context of the MSSM except the Higgs which is also part of a chiral multiplet but this time ($\psi$, $\phi$)

\[ H_{33} \rightarrow \xi \uparrow H_{33} \]

You may ask: What about gravity? Well normally any discussion of the SM ignores gravity

(set in IM flat spacetime).

But if we get fussy and consider gauging this new transformation, then according to:

\[ \{ Q_{\lambda}, Q_{\lambda}\} = 2 \delta_{\lambda} \quad \text{and} \quad \quad \text{making these local means making} \quad S \quad \text{local} \]

But a theory of gauged \(1+1\)-translations is exactly GR!

\[ \downarrow \text{graviton} \quad \downarrow \text{gravitino} \]

So gauging SUSY $\rightarrow$ SU(2) $\rightarrow$ GR! and we then get to add $\left( \mathcal{G}_{\mu \nu}, \frac{1}{2} R \right)$ to the mix.
What does SUSY give us?

As discussed before, the SM couplings almost cross at a common value under renormalization. Adding SUSY makes them meet at the same point, which strongly hints at unification of $SU(3)\times SU(2)\times U(1)$.

Moreover, if we consider the renormalization of the Higgs mass (the only fundamental mass in the SM) it should get corrections up to some unification scale, but this would make it too large (it is at the EW scale). Fortunately, adding SUSY has the effect that SM contributions to the running of the Higgs mass are cancelled by contributions from the superpartners. SUSY solves the hierarchy problem.

What about SUSY? If we add naive perturbative QCD to the SM, we encounter divergences which cannot be renormalized (the theory is basically scale).

Adding SUSY does provide some cancellations, but ours still abound.