Tuesday, November 6, 2018
7:35 AM

Theories of Everything

GUTs

$$\underbrace{\text{SM ☺} \qquad \text{Gravity ☹}}_{\text{TOE}}$$

$\Downarrow$

SM + Gravity ☺

How can we get gravity in the game? Well the first thing to remember is that the forces in the SM are all based on gauge symmetry principles.

EM: $U(1)$ complex phase rotations of spinors

Weak: $SU(2)_L$ "rotations" of complex doublets $\binom{u}{d}_L$, $\binom{e}{\nu_e}_L$, etc.

Strong: $SU(3)$ rotations of complex color triplets $\begin{pmatrix} r \\ b \\ g \end{pmatrix}$

coefficients in linear combinations are complex

Is gravity based on a symmetry? Yes!

Full GR is based on arbitrary coordinate invariance, or $GL(\overset{D}{\overset{\shortparallel}{4}}, \mathbb{R})$.

general linear real coeff.

That is $S = \int d^4x \sqrt{-\det g}\, R$ coord. invariant

or $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$ coord. covariant

Thus coordinate transformations are gauge symmetries (or redundancies) just like the $SU(3) \times SU(2)_L \times U(1)$ of the SM.

So what is the difference? $GL(4, \mathbb{R})$ acts on spacetime, while $SU(3) \times SU(2)_L \times U(1)$ acts on internal or iso-spaces.

But what if we just made spacetime larger by including the iso-spaces as real spaces?

It turns out the math works out beautifully, and almost perfectly. In fact the idea is so slick and straightforward that the earliest version was proposed shortly after GR (when EM was the only other well understood force.)

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Consider GR in 5D, so $S \propto \int d^5x \sqrt{-\det G}\, R_G$ w/ $G_{ab} = \left(\begin{array}{c|c} g_{\mu\nu} & g_{\mu 5} \\ \hline g_{5\nu} & g_{55} \end{array}\right)$

$a,b \in (0,1,2,3,5)$

$\mu,\nu \in (0,1,2,3)$

If we assume independence on x^5, i.e. $g_{\mu\nu}(x^\mu), g_{5\nu}(x^\mu), g_{\mu 5}(x^\mu), g_{55}(x^\mu)$ then we find:

$$S \propto \int d^4x \sqrt{-\det g}\left(R_g - \underbrace{F_{\mu\nu}}_{\partial_\mu A_\nu - \partial_\nu A_\mu} F^{\mu\nu} + \partial_\mu g_{55}\, \partial^\mu g_{55}\right) \quad (\text{some coefficients are} \neq 1)$$

$= \underbrace{GR + EM}_{\ddot\smile} +$ scalar field

A few questions remain:

1) Why independence of x^5? Well make $\underbrace{x^5 = x^5 + L}$ w/ L small, then QM says $p_5 \propto \frac{n}{L}$ $n = 0,1,2\cdots$

Secretly this is just a circle so spacetime is $W^{1+3} \times S^1$

and for L really small, p_1 is beyond observed energies.

2) What about $SU(3) \times SU(2)$? We know that we can get $U(1)$ from adding a 1D circle to spacetime.

For $SU(2)$ the smallest space we can add is S^2 ($SO(3) \sim SU(2)$ isometries)

For $SU(3)$ the smallest space is CP^2, i.e. $(\underbrace{x,y,z}_{\text{all complex and } \neq 0}) \sim (\lambda x, \lambda y, \lambda z)$ $\}$ any real # (+ or −)

CP^2 is 4 real dimensional

So to get all of $SU(3) \times SU(2) \times U(1)$ from KK we need $4 + 4 + 2 + 1 = 11$ spacetime dimensions

That turns out to be a very special number of dimensions for reasons to be discussed.

But, Witten poo-pooed the whole program by pointing out that KK gives $SU(2)$, not $SU(2)_L$!

However bigger problems loomed:

$$\text{SM} + \text{Gravity}\ \ddot{\smile} \ \overset{+\text{QM}}{\Longrightarrow} \ \text{SM}\ \ddot{\smile} \quad \text{Gravity}\ \ddot{\frown}$$

What is the problem? Early in the days of QFT, renormalizability was a criterion for the acceptedness of a theory. In a nutshell, a renormalizable theory is one for which ∞'s (QFTs almost always have them) can be attributed to our working in terms of bare (unscreened) values of masses, charges, etc. instead of the more physical, i.e. measured, values.

One of the triumphs of the SM was the demonstration of its renormalizability.
One of the epic failures of $\underbrace{\text{naive quantum gravity}}_{\text{a QFT}}$ is its nonrenormalizability.

But times have changed... with Wilson's insights into effective field theories, we now appreciate nonrenormalizable theories as those which predict their own limitations.

So the modern understanding is that naive quantum gravity is useful up to some high energy scale, beyond which it should be replaced by a theory w/ different degrees of freedom.