Review: must be normalized

Discrete: $\sum_{n} c_n \psi_n(x) \quad c_n = \frac{\langle \psi_n | x \rangle}{\int |\psi_n(x)|^2 dx}$

$\phi_0(x,t) \quad \psi_n(x,t)$

$\left\{ \right\}$ TISE (WKB)

Continuous: $E(\psi_0) = \frac{1}{2m} \int |\psi(x)|^2 \dot{\psi}(x) dx$

$\phi_{\psi}(x,t) = \frac{1}{2\pi \hbar} \int \psi(y) \exp(-i\frac{\hbar y}{\sqrt{2m} \psi(x)}} dx$

free particle: $\psi(x,t) = \exp(-i\frac{\hbar x^2}{2m} t)$

(ground states)

Discrete - arise when motion is bounded (can not visit $x \rightarrow \infty$)

Continuous - arise when motion is unbounded (can at least approach)

(Scattering States)

Can we get both?

Bound

Scattering

Classical

(Only for small)

(Always V=0)

Quantum

(Always)

(Always V)

Scattering

Only V states

$S \& B$ states

$S \& B$ states

To determine the states, classical requires detailed info.

In QM, all we need is how E and configuration V($\pm \infty$).

$E < V(\pm \infty)$

$E > 2V(\pm \infty)$

$E > 2V(\pm \infty)$

We are usually given $V(\pm \infty) = 0$!

$E < 0 \quad B \quad E > 0 \quad S$

$E > 0 \quad S \quad E > 0 \quad S$

$E < 0 \quad B$
General Approach

1. In TESE $v, E_n$ in each region.
2. Write $v, E_n$ at each region.
3. $E_n + v_n = 0$
4. $E_n$ and continuity $E_n + E_n = 0$ at some location

$E_n(0) = E_n(1) = k$ = constant

$E_n(x) = k \sin x$

$E_n(0) = E_n(1) = k = A_k$

Today's Note: (Note: $E_n < 0$)

$x > 0$ $E_n = \frac{d^2 v}{dx^2}$

$x = 0$ $v(0) = 0$

So far we have known $v_0, b_0, A_k$

We cannot solve.

Continuity: $v^+(x = 0) = v^-(x = 0)$

$b_0 = A_k = A_k \
\Rightarrow b_1 = A_k$