\[
\begin{align*}
\text{Scattering States: } & \quad \psi(x) = e^{ikx} + e^{-ikx} \\
\text{S-wave: } & \quad I | \quad \text{II} \\
\end{align*}
\]

\[
E = \frac{1}{2} m \omega^2 r^2.
\]

\[
\text{Scattering: } E > 0 \\
T \psi(x) = \frac{1}{2} m \omega^2 r^2 \Rightarrow \psi(x) = A e^{ikx} + B e^{-ikx}
\]

\[
\begin{align*}
\psi(x) & = A e^{ikx} + B e^{-ikx} \\
\psi(0) & = A e^{i0} + B e^{-i0} \\
\end{align*}
\]

\[
\text{Finally, we need } \psi(0) = 0. \text{ But not this time!}
\]

5 unknowns: \(A, B, k, E, (E)
\]

\[
\begin{align*}
\text{Constant: } & \quad C_{\text{const}} = \psi(0) \quad A + B = C + E \\
\text{Scattering: } & \quad \frac{d}{dx} \psi(x) = -ik \psi(x) \\
\text{Scattering: } & \quad \psi(0) = A e^{i0} + B e^{-i0} \\
\text{Scattering: } & \quad \psi(0) = A(1) + B(-1) \\
\text{Scattering: } & \quad ik(1) - i(k(1) - k(1)) = -\frac{1}{2} E^2 (A + B)
\end{align*}
\]

2 equations, 5 unknowns ... problem!!

Due Tuesday:

What do we expect for \(k^2\)? \(\frac{1}{k^2} = \leq k^4\)

5 unknowns is unique

Typical (bound) \(k\) is discrete, e.g. \(k = \frac{\pi}{L}\)

Typical (scattering) \(k\) is continuous

K is an eigenvalue

a) What about \(A, B, C, E\)? We going to find a physical interpretation for these and then formulate the problem.

b) Then leaves 3 unknown, 2 equations: Can you find a solution?

Look for notes.
We will assume approach from left, $x = 0$

(I) unknowns: $A$, $B$  
For an incident wave, how much is transmitted and how much is reflected? (R)

$R = \frac{1 + k}{2}$  
$T = \frac{1 - k}{2}$

Expected: $R + T = 1$

How to find $R$, $T$:

1. $A = 3 + 3i$  
2. $T = \frac{1 + 3i}{2}$

Center to mean:

$A = \frac{3 - 3i}{2}$  
$B = \frac{3 + 3i}{2}$

$R = \frac{2}{1 + i} = \frac{1 - i}{2}$

$T = \frac{3}{1 + i} = \frac{1 + i}{2}$

Conclusion:

$B = \frac{1}{2} A$

Center expression:

$B(E) = \frac{A - \bar{E}}{k}$  
$A = \frac{1}{\alpha k}$

$R = \frac{\alpha k}{\alpha k + 1}$  
$T = \frac{1}{1 + \frac{\alpha k}{\alpha k + 1}}$

$\lim_{k \to 0} R = 0$  
$\lim_{k \to 0} T = 1$
Consider scattering from left: \( A_1, C_1, D_1, B_1, F \)

4 constraints: \( C(-a), C(a) \)
\( S(-a), S(a) \)

\[
T = \left( \frac{E_1^2}{1\text{r}_1^2} \right) \left[ 1 + \frac{V_0^2}{4E_0(E_0+\omega)} \sin^2 \left( \frac{\pi \omega}{E_0} \right) \right]^{-1}
\]

\[
R = \frac{10E_3^2}{1\text{r}_1^2}
\]

As \( E \to \infty \) \( V_0 = 0 \) \( \Rightarrow T \to 1 \)

As \( V_0 \to \infty \) \( \text{fixed} \) \( E \to T \to 0 \)

\[
\omega \to \frac{1}{E_0} \left( E_0 + \omega_0 \right) = \text{const} \Rightarrow T = 1
\]