Two-particle System

\[ H_{\text{tot}} = -\frac{k_1}{2\hbar^2} \nabla_1^2 - \frac{k_2}{2\hbar^2} \nabla_2^2 + V(r_1, r_2, t) \]

\[ \sum \psi^*(\vec{r}_1, \vec{r}_2, t) \psi(\vec{r}_1, \vec{r}_2, t) \, d^3r_1 \, d^3r_2 = 1 \]

TISE: \[ \text{i} \hbar \frac{\partial}{\partial t} \psi = \hat{H}_{\text{tot}} \psi \]
\[ \text{i} \hbar \frac{\partial}{\partial t} \psi = \hat{H}_1 \psi \]
\[ \text{i} \hbar \frac{\partial}{\partial t} \psi = \hat{H}_2 \psi \]

Assume general solution expands in a basis of separable solutions (stationary states): \[ \psi(\vec{r}_1, \vec{r}_2, t) = \sum \psi_n(\vec{r}_1, \vec{r}_2) e^{-\text{i}Et} \]

Assume discrete (we keep for cont.)

Where \( \psi_n(\vec{r}_1, \vec{r}_2) \) satisfy TISE: \[ \hat{H}_{\text{tot}} \psi_n(\vec{r}_1, \vec{r}_2) = E_n \psi_n(\vec{r}_1, \vec{r}_2) \]

form an orthonormal set
Is that all? No!!!

a product: 1, 2
Each of these must be in some state: \( \psi_a \) or \( \psi_b \)
We might write: \( \psi_a \psi_b = \psi_a \psi_b \psi_a \psi_b \)

If 2 particles on the same state, what is the effect of swapping them?

\( \psi_{12} \psi(\bar{\psi}_a, \bar{\psi}_b) = \psi(\bar{\psi}_b, \bar{\psi}_a) \)

\( \psi_{12} \psi(\bar{\psi}_a, \bar{\psi}_b) = \psi_{12} \psi(\bar{\psi}_b, \bar{\psi}_a) \)

\( \psi_{12} \) has eigenvalue \( \pm 1 \)

How is \( \psi(\bar{\psi}_a, \bar{\psi}_b) \) related to \( \psi(\bar{\psi}_b, \bar{\psi}_a) \)?

\( +1 : \psi_{12} \psi(\bar{\psi}_a, \bar{\psi}_b) = +1 \psi(\bar{\psi}_b, \bar{\psi}_a) = \psi(\bar{\psi}_b, \bar{\psi}_a) \)

\( -1 : \psi_{12} \psi(\bar{\psi}_a, \bar{\psi}_b) = -1 \psi(\bar{\psi}_b, \bar{\psi}_a) = \psi(\bar{\psi}_b, \bar{\psi}_a) \)

\( \psi_+ = \frac{1}{\sqrt{2}} \left( \psi_a(\bar{\psi}_a) + \psi_b(\bar{\psi}_a) \right) \) \( \psi_- = \frac{1}{\sqrt{2}} \left( \psi_a(\bar{\psi}_b) - \psi_b(\bar{\psi}_b) \right) \)

\( \psi_+ \psi_+ = \frac{1}{4} \left( \psi_a(\bar{\psi}_a) \psi_a(\bar{\psi}_b) + \psi_b(\bar{\psi}_a) \psi_b(\bar{\psi}_b) \right) \)

\( \psi_- = \frac{1}{\sqrt{4}} \left( \psi_a(\bar{\psi}_b) - \psi_b(\bar{\psi}_b) \right) \) \( \psi_- = \frac{1}{4} \left( \psi_a(\bar{\psi}_a) \psi_a(\bar{\psi}_b) - \psi_b(\bar{\psi}_a) \psi_b(\bar{\psi}_b) \right) = 0 \)