Problem 2.23

(a) 
\[ (-2)^3 - 3(-2)^2 + 2(-2) - 1 = -8 - 12 - 4 - 1 = -25. \]

(b) 
\[ \cos(3\pi) + 2 = -1 + 2 = 1. \]

(c) 
\[ 0 \] (x = 2 is outside the domain of integration).
Problem 2.27

(a) 

(b) From Problem 2.1(c) the solutions are even or odd. Look first for even solutions:

\[ \psi(x) = \begin{cases} 
    Ae^{-\kappa x} & (x < a), \\
    B(e^{\kappa x} + e^{-\kappa x}) & (-a < x < a), \\
    Ae^{\kappa x} & (x < -a). 
\end{cases} \]

Continuity at \( a \): \( Ae^{-\kappa a} = B(e^{\kappa a} + e^{-\kappa a}) \), or \( A = B(e^{2\kappa a} + 1) \).

Discontinuous derivative at \( a \), \( \Delta \frac{d\psi}{dx} = -\frac{2m\alpha}{\hbar^2} \psi(a) \):

\[ -\kappa Ae^{-\kappa a} - B(\kappa e^{\kappa a} - \kappa e^{-\kappa a}) = -\frac{2m\alpha}{\hbar^2} A e^{-\kappa a} \Rightarrow A + B(e^{2\kappa a} - 1) = \frac{2m\alpha}{\hbar^2} A; \text{ or} \]

\[ B(e^{2\kappa a} - 1) = A \left( \frac{2m\alpha}{\hbar^2 \kappa} - 1 \right) = B(e^{2\kappa a} + 1) \left( \frac{2m\alpha}{\hbar^2 \kappa} - 1 \right) \Rightarrow e^{2\kappa a} - 1 = e^{2\kappa a} \left( \frac{2m\alpha}{\hbar^2 \kappa} - 1 \right) + \frac{2m\alpha}{\hbar^2 \kappa} - 1. \]

This is a transcendental equation for \( \kappa \) (and hence for \( E \)). I'll solve it graphically: Let \( z = 2\kappa a \), \( c = \frac{\hbar^2}{2ma} \), so \( e^{-z} = cz - 1 \). Plot both sides and look for intersections:
From the graph, noting that \( c \) and \( z \) are both positive, we see that there is one (and only one) solution (for even \( \psi \)). If \( \alpha = \frac{h^2}{2ma} \), so \( c = 1 \), the calculator gives \( z = 1.278 \), so \( \kappa^2 = -\frac{2mc}{h^2} = \frac{z^2}{(2a)^2} \Rightarrow E = -\frac{(1.278)^2}{8} \left( \frac{h^2}{ma} \right) = -0.204 \left( \frac{h^2}{ma} \right) \).

Now look for odd solutions:

\[
\psi(x) = \begin{cases} 
    Ae^{-\kappa x} & (x < a), \\
    B(e^{\kappa x} - e^{-\kappa x}) & (-a < x < a), \\
    -Ae^{\kappa x} & (x < -a).
\end{cases}
\]

Continuity at \( a \): \( Ae^{-\kappa a} = B(e^{\kappa a} - e^{-\kappa a}) \), or \( A = B(e^{2\kappa a} - 1) \).

Discontinuity in \( \psi' \): \( -\kappa Ae^{-\kappa a} - B(\kappa e^{\kappa a} + \kappa e^{-\kappa a}) = -\frac{2m\alpha}{h^2} Ae^{-\kappa a} \Rightarrow B(e^{2\kappa a} + 1) = A \left( \frac{2m\alpha}{h^2} - 1 \right) \),

\[
e^{2\kappa a} + 1 = (e^{2\kappa a} - 1) \left( \frac{2m\alpha}{h^2} - 1 \right) = e^{2\kappa a} \left( \frac{2m\alpha}{h^2} - 1 \right) - \frac{2m\alpha}{h^2} + 1,
\]

\[
1 = \frac{2m\alpha}{h^2} - 1 - \frac{2m\alpha}{h^2} e^{-2\kappa a} \frac{h^2 K}{m\alpha} = 1 - e^{-2\kappa a}, \quad \frac{h^2 K}{m\alpha} = 1 - e^{-2\kappa a} = 1 - \frac{h^2 K}{m\alpha}, \text{ or } e^{-z} = 1 - cz.
\]

This time there may or may not be a solution. Both graphs have their \( y \)-intercepts at 1, but if \( c \) is too large (\( \alpha \) too small), there may be no intersection (solid line), whereas if \( c \) is smaller (dashed line) there will be. (Note that \( z = 0 \Rightarrow \kappa = 0 \) is not a solution, since \( \psi \) is then non-normalizable.) The slope of \( e^{-z} \) (at \( z = 0 \)) is \(-1\); the slope of \((1 - cz)\) is \(-c\). So there is an odd solution \( \Leftrightarrow c < 1 \), or \( \alpha > h^2/2ma \).

Conclusion: One bound state if \( \alpha < h^2/2ma \); two if \( \alpha > h^2/2ma \).