Using Integer Programming for Strategic Underground and Open Pit-to-Underground Scheduling

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Cutoff Grade – Introduction

- Mining companies must make the distinction between ore and waste material, *cutoff grade*
- Cutoff grade signifies the degree of mineralization below with material will not be processed into a salable product
- Our research determines the cutoff grade that maximizes the NPV of a planned underground mine
Cutoff Grade – Mine Design

- Stopes are constructed on a regular grid
- Width and length of a stope may vary within a fixed minimum and maximum value
- Height of the stopes is equal on every level
The mine is separated into mining zones and each mining zone may possess its own cutoff grade.
Cutoff Grade – Mine Design

**Sequencing Rules:**

1. Stopes in the left corridor are extracted increasing by slot number and stopes in the right corridor are extracted decreasing by slot number.

2. If multiple stopes exist at the same level-slot location, they are extracted in order of economic value.

3. A stope may not be extracted unless the stope(s) directly above have been completely extracted.
We enumerate all cutoff-grade triples, which we define as a permutation of the 16 cutoff grade options, 1 to 7 by 0.4, for the South, Central, and North Zone.

- Fast solutions times for each cutoff-grade triple
  - The OMP Solver (Rivera et al., 2015) can quickly solve the linear programming relaxation of the UG-PSP strategic formulation
  - TopoSort can create near-optimal integer solutions to UG-PSPs in seconds (Chicoisne et al., 2012; King et al., 2016b)

- Ability to bound the objective function value for each solve
  - Linear programming relaxation objective function value provides an upper bound
  - Integer solution objective function value provides a lower bound
Creating an integer programming solution to the linear programming relaxation with the largest objective function value results in a schedule that cannot be implemented.

- Cutoff grades are 3.8, 3.4, and 3.0 units per tonne for the South, Central, and North Zones, respectively.
- Integer Programming gap of 0.11% and an NPV of 85.53
We adjust the resource constraints to prevent the North Zone from extracting stopes until year 9, and this results in more desirable production levels.

- Cutoff grades remain at 3.8, 3.4, and 3.0 units per tonne for the South, Central, and North Zones, respectively.
- NPV is only 1.8% lower than that of the Original Schedule.
• Cutoff-grade triple instances, on average, contain 31,575 variables and 230,117 constraints
• Linear programming relaxation solution time of each cutoff-grade triple averages 8.10 seconds
  ○ Minimum and maximum solution times are 1.59 and 19.02 seconds, respectively
• Parallel computation allows us to solve all of 4,096 linear programming relaxations in 43 minutes
Cutoff Grade – Conclusions

- Outline a methodology for producing the optimal cutoff grade for multiple mining zones by using an integer program.
- Exploit the mathematical structure of the problem to solve the linear programming relaxation efficiently using the OMP Solver.
- Provide three distinct improvements to the current method for selecting the cutoff grade:
  1. Determines an upper bound on NPV as opposed to using a heuristic.
  2. Updating the data and rerunning of our model requires a few days, contrary to the six to eight weeks it takes to rerun the mining company’s model.
  3. Schedules the entire mine and selects the optimal cutoff grade for multiple zones rather than considering only a single zone.
Mineral deposits may extend across a large vertical expanse.
Mining industry desires a methodology to determine which areas of the deposit to extract via open pit or underground mining methods.
Mine planners are forced to schedule the open pit and underground mines separately.
Open pit is separated into (aggregated) blocks
Blocks contain bins of (i) waste, (ii) low-, (iii) medium-, and (iv) high-grade ore
Each phase-block-bin combination has its own stockpile
Material from each bin can be sent to the mill, to a stockpile, or to a waste location
“Standard” Inventory Balance Constraint

Material in the stockpile at the start of the next time period =
Material currently in the stockpile - Material sent from stockpile to
mill + Material added to the stockpile

\[ I_{nb,t+1}^S = I_{nbt}^S - I_{nbt}^S + (Y_{nb2t}^S - Y_{nb2,t-1}^S) \quad \forall b \in B, n \in \mathcal{N}_b, t \in \mathcal{T} \]
We can reformulate the stockpiling to precedence relationship by placing all of the material in a stockpile before processing.

- Equivalent to the original constraint under the assumptions:
  - No value lost for placing material in inventory
  - No handling costs associated with placing or retrieving material

Reformulated Constraint

Amount of Material Sent to Processing $\leq$ Amount of Material Extracted

$Z_{nbt}^S \leq \hat{Y}_{bt}^S \quad \forall b \in B, n \in N_b, t \in T$
Open stope mining method using Modified-Avoca sequencing

Stopes are constructed in a regular grid

Development position and length are estimated
A crown pillar is a layer of rock left between the bottom of the open pit and the top of the underground mine for safety.
A sill pillar is a level of stopes left in situ to provide a false bottom in an underground mine.
Levels are horizontal expanses of rock whose height is equal to a maximum height of the stope.
**Indices and sets:**

- $v \in \mathcal{V}$ set of crown pillar elevations
- $\tilde{b} \in \tilde{\mathcal{B}}_v$ set of blocks that exist below the crown pillar if the crown pillar is located at elevation $v$
- $\tilde{a} \in \tilde{\mathcal{A}}_v$ set of activities that exist above the crown pillar if the crown pillar is located at elevation $v$

**Decision variables:**

- $W_{Tb}^v$ \[ W_{Tb}^v = 1 \text{ if the crown pillar is located at elevation } v; \ 0 \text{ otherwise} \]
- $X_{Ut}^a$ \[ X_{Ut}^a = 1 \text{ if activity } a \text{ is completed by the end of time } t; \ 0 \text{ otherwise} \]
- $X_{St}^{bt}$ \[ X_{St}^{bt} = 1 \text{ if block } b \text{ is completed by the end of time } t; \ 0 \text{ otherwise} \]

**Constraints:**

- $X_{St}^{bt} \leq 1 - W_{Tb}^v \quad \forall \tilde{b} \in \tilde{\mathcal{B}}_v, v \in \mathcal{V}, t \in \mathcal{T}$
- $X_{Ut}^a \leq 1 - W_{Tb}^v \quad \forall \tilde{a} \in \tilde{\mathcal{A}}_v, v \in \mathcal{V}, t \in \mathcal{T}$
- $\sum_{v \in \mathcal{V}} W_{Tb}^v = 1$
Enumerating crown and sill pillar locations allows us to remove the “or” precedence constraints whose presence drastically increases the solve time.
Transition – Model

- **Variables:**
  - Open pit block extraction time period
  - Open pit block processing time period
  - Underground activity completion time period

- **Objective Function:** Maximize NPV

- **Constraints:**
  - Blocks and activities can be completed once
  - Precedence constraints for both the open pit and underground mine are maintained
  - Capacity constraints for the open pit mine, underground mine, and processing plant are met
Summary of annual production capacities for the operational metal mine

<table>
<thead>
<tr>
<th>Annual Capacities</th>
<th>Annual Discount Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extraction</td>
<td>Development</td>
</tr>
<tr>
<td>Open Pit (t)</td>
<td>Underground (t)</td>
</tr>
<tr>
<td>50,000,000</td>
<td>2,000,000</td>
</tr>
</tbody>
</table>

Crown pillar located at an elevation of 820 meters above sea level and a sill pillar located at level 460

<table>
<thead>
<tr>
<th>CPLEX (12.6)</th>
<th>OMP Solver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrier Time (sec)</td>
<td>Simplex Time (sec)</td>
</tr>
<tr>
<td>163.75</td>
<td>488.41</td>
</tr>
</tbody>
</table>

Models are solved on a Dell PowerEdge R410 machine with 16 processors (2.72 GHz each) and 28 GB of RAM
Transition – LP and IP Enumeration Results

LP Relaxation Objective Function Values

Best Known IP Objective Function Values

NPV (scaled)

Crown Pillar Elevation

Crown Pillar Elevation
Best known IP objective function value had a 2.51% gap

Crown pillar placement was at elevation 820 and a sill pillar located on level 500

Serial solution time to solve over 3500 LP relaxations and to obtain a near-optimal IP solution in 8 hours and 15 minutes

Fully parallelized the problem can be solved in 39.75 seconds
Transition Model: Indices and sets

\[ b \in \mathcal{B} \quad \text{blocks } b \]
\[ n \in \mathcal{N}_b \quad \text{bins in block } b \]
\[ \hat{b} \in \hat{\mathcal{B}}_b \quad \text{blocks that must be mined directly before block } b \]
\[ \tilde{p} \in \tilde{\mathcal{P}}_b \quad \text{predecessors for block } b \text{ that must be completed at least one time period in advance of block } b \]
\[ a \in \mathcal{A} \quad \text{set of all underground activities} \]
\[ \check{a} \in \check{\mathcal{A}}_a \quad \text{set of fixed predecessors for activity } a \]
\[ \bar{p} \in \bar{\mathcal{P}}_a \quad \text{predecessors for activity } a \text{ that must be completed at least one time period in advance of activity } a \]
\[ r \in \mathcal{R} \quad \text{resources (1 = open pit mine, 2 = mill, 3 = sinking rate, 4 = underground mine capacity, 5 = backfill capacity, 6 = development capacity)} \]
\[ t \in \mathcal{T} \quad \text{time periods} \]
Transition Model: Data

$c_{nb}^{S-}$ mining cost for bin $n$ in block $b$ [$/tonne]

$c_{nb}^{S+}$ revenue generated after having milled bin $n$ of block $b$ [$/tonne]

$q_{rnb}^{S}$ quantity of resource $r$ consumed by bin $n$ of block $b$ [1 & 2 = tonnes]

$c_{a}^{U}$ monetary value associated with completing activity $a$ [$]

$q_{ra}^{U}$ quantity of material requiring extraction associated with activity $a$ [2, 4 & 5 = tonnes, 6 = meters]

$r_{rt}, \bar{r}_{rt}$ minimum, maximum amount of resource $r$ available in time $t$ [1 & 2 = tonnes, 3 = blocks, 4 & 5 = tonnes, 6 = meters]

$\delta_{t}$ discount factor for time period $t$ (fraction)
Decision Variables

\( X_{bt} \) 1 if block \( b \) has finished being extracted by the end of time \( t \); 0 otherwise

\( \hat{Y}_{bt} \) fraction of block \( b \) extracted and sent to the stockpile by the end of time \( t \)

\( Z_{nbt} \) fraction of bin \( n \) in block \( b \) sent to the mill by the end of time \( t \)

\( X_{at} \) 1 if activity \( a \) is completed by the end of time \( t \); 0 otherwise

- **Objective Function: Maximize Net Present Value**

\[
\max \sum_{b \in B} \sum_{n \in N_b} \sum_{t \in T} \delta_t c_{nb}^S q_{2nb}^S (Z_{nbt}^S - Z_{nb,t-1}^S) - \\
\sum_{b \in B} \sum_{t \in T} \delta_t c_{b}^S q_{1nb}^S (\hat{Y}_{bt}^S - \hat{Y}_{b,t-1}^S) + \sum_{a \in A} \sum_{t \in T} \delta_t c_a^U (X_{at}^U - X_{a,t-1}^U)
\]
Transition Model: Constraints

- Ensure that each completed activity or block remains completed

\[
\begin{align*}
X^S_{b,t-1} &\leq X^S_{bt} \quad \forall b \in B, t \in T \\
\hat{Y}^S_{b,t-1} &\leq \hat{Y}^S_{bt} \quad \forall b \in B, t \in T \\
Z^S_{nb,t-1} &\leq Z^S_{nbt} \quad \forall b \in B, n \in N_b, t \in T \\
X^U_{a,t-1} &\leq X^U_{at} \quad \forall a \in A, t \in T 
\end{align*}
\]

- Enforce the precedence structure

\[
\begin{align*}
X^S_{bt} &\leq \hat{Y}^S_{bt} \quad \forall b \in B, t \in T \\
\hat{Y}^S_{bt} &\leq X^S_{\hat{p}t} \quad \forall b \in B, \hat{p} \in \hat{P}_b, t \in T \\
Z^S_{nbt} &\leq \hat{Y}^S_{bt} \quad \forall b \in B, n \in N_b, t \in T \\
X^U_{at} &\leq X^U_{\hat{p}t} \quad \forall a \in A, \hat{p} \in \hat{P}_a, t \in T 
\end{align*}
\]
Transition Model: Constraints

- Ensure that one time period elapses between the completion of two specific activities or blocks
  \[
  \hat{Y}_{bt}^S \leq \hat{Y}_{\tilde{p}, t-1}^S \quad \forall b \in B, \tilde{p} \in \tilde{P}_b, t \in \mathcal{T}
  \]
  \[
  X_{at}^U \leq X_{\tilde{p}, t-1}^U \quad \forall a \in A, \tilde{p} \in \tilde{P}_a, t \in \mathcal{T}
  \]

- Bounds open pit-specific resource use
  \[
  r_{1t} \leq \sum_{b \in B} \sum_{n \in \mathcal{N}_b} q_{1nb}^S (\hat{Y}_{bt}^S - \hat{Y}_{b, t-1}^S) \leq \bar{r}_{1t} \quad \forall t \in \mathcal{T}
  \]

- Restricts the mill capacity
  \[
  r_{2t} \leq \sum_{b \in B} \sum_{n \in \mathcal{N}_b} q_{2nb}^S (Z_{nb}^S - Z_{nb, t-1}^S) + \sum_{a \in A} q_{2a}^U (X_{at}^U - X_{a, t-1}^U) \leq \bar{r}_{2t} \quad \forall t \in \mathcal{T}
  \]

- Bounds underground-specific resource consumption
  \[
  r_{rt} \leq \sum_{a \in A} q_{ra}^U (X_{at}^U - X_{a, t-1}^U) \leq \bar{r}_{rt} \quad \forall r \in \mathcal{R}, r \geq 4, t \in \mathcal{T}
  \]