# Truthful Auction for Cooperative Communications with Revenue Maximization

Dejun Yang, Xi Fang, and Guoliang Xue

*Abstract*—**Auction theory has been applied to cooperative communications to either efficiently allocate resources or incentivize wireless devices to participate in cooperative communications. However, a common shortcoming of the existing studies is that the revenue generation is neglected. Revenue generation is the ultimate goal of commercial networks, e.g., WiMAX networks. In this paper, we study the problem of how to use auction mechanisms to allocate the relay nodes and charge the source nodes, such that the revenue of the seller, e.g., the base station, is maximized. We first propose a VCG-based auction mechanism, which can maximize the revenue while enforcing the truthfulness. To overcome the high time complexity of the VCG-based auction mechanism, we further design another truthful auction mechanism with low time complexity. Experiment results show that the suboptimal auction mechanism significantly reduces the time complexity without severely sacrificing the revenue.**

## I. Introduction

Recently, cooperative communication has attracted a surge of interest. Aiming to achieve spacial diversity, cooperative communication exploits the broadcast nature of wireless transmissions and takes advantage of the antennas on wireless devices. Depending on how the relay node processes the signal received from the source node, there are two widely used cooperative communication modes, *decode-and-forward* and *amplify-and-forward* [6].

Auction-based mechanisms have been used to efficiently allocate resources [3, 8, 9, 11] or motivate independent individuals to participate in cooperative communications [9, 10]. A common shortcoming of the existing studies on the auction design for cooperative communications is that they neglect the ultimate goal of the service companies, which is to maximize their revenue. Take the WiMAX network as an example, as shown in Fig. 1. According to the IEEE 802.16$j$ standard [4], Relay Stations (RS) have been introduced to relay traffic for Subscriber Stations (SSs). Since these RSs belong to the service company, motivating the relay nodes (RSs in this case) to participate in cooperative communications is no longer a requirement while designing an auction mechanism.

**Our focus in this paper is on the design of revenue-maximizing auction mechanisms for cooperative communications.** In the auction, we allow each source node to bid for a combination of relay nodes. This auction is known as a *combinatorial auction*, where bidders can express their valuations of combinations of items.

All authors are affiliated with Arizona State University, Tempe, AZ 85287. E-mail: {dejun.yang, xi.fang, xue}@asu.edu. This research was supported in part by ARO grant W911NF-09-1-0467 and NSF grant 0905603. The information reported here does not reflect the position or the policy of the federal government.

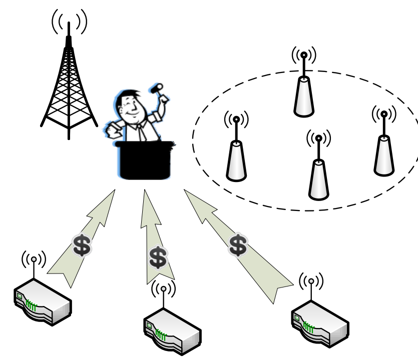


Fig. 1. Auction

Main Contributions: We first propose a VCG-based truthful auction mechanism, which guarantees to maximize the revenue. To overcome the high time complexity of the VCG-based auction mechanism, we then design a suboptimal truthful auction mechanism, which significantly reduces the time complexity without severely sacrificing the revenue.

We organize the remaining of this paper as follows: In Section II, we briefly review the existing studies on auction mechanism design for cooperative communications. In Section III, we introduce the network model considered in this paper, describe the multi-relay cooperative communication model, and give the auction formulation. To maximize the revenue from the auction, we first propose a VCG-based auction mechanism in Section IV. We then design a suboptimal auction mechanism with low time complexity in Section V. Before concluding this paper in Section VII, we evaluate the performance of designed auction mechanisms in Section VI.

## II. Related Work

Auction theory has been studied for decades. However, in the literature, limited work has been done on the design of auction mechanisms for cooperative communications. Among them are [3, 8–11].

In [8], Wang *et al.* formulated the trade between a source node and multiple relay nodes as a buyer's market, and thus modeled it as a Stackelberg game with the source node as the leader and the relay nodes as the followers. Zhang *et al.* studied a different scenario, where there is one relay node and multiple source nodes [11]. The market becomes a seller's market. The strategy of each source node is the bandwidth it wants to buy. A distributed algorithm was developed to search a Nash Equilibrium (NE). In [3], the relay node is the seller and the source nodes are bidders, who submit bids to the relay node. The relay node then allocates its transmission power proportional to the source nodes' bids. The existence and the uniqueness of the NE were proved. In addition, it was proved that the distributed best response bid updates could converge globally to the unique NE in a completely asynchronous manner.

The above work only considered the selfish behavior of players, but ignored the *cheating* behavior. It has been shown both theoretically and practically that a market could be vulnerable to market manipulation and produce very poor outcomes if players are dishonest on their valuations, e.g., prices. Therefore truthfulness is the most critical property of mechanism design. In [10], Yang *et al.* designed an auction scheme for cooperative communications, which satisfies not only truthfulness, but also individual rationality and budget balanced properties. From the system's perspective, they developed payments to induce the source nodes to select the relay nodes, such that the system performance is optimized, and to influence the relay nodes to report their valuations truthfully.

A shortcoming of the above work is that the revenue factor has been neglected. In commercial networks, like WiMAX networks, the relay nodes are owned by the same entity. The ultimate goal of the service companies for building the relay stations is to maximize their revenue. To fill this void, we focus on designing auction mechanisms for cooperative communications, with the objective of revenue maximization.

## III. System Model and Auction Formulation

### A. Network Model

We consider a static network consisting of $n$ source nodes (SNs), denoted by $\mathcal{S} = \{s_1, s_2, \ldots, s_n\}$, and $m$ relay nodes (RNs), denoted by $\mathcal{R} = \{r_1, r_2, \ldots, r_m\}$. We assume that the SNs have the common destination node $d$, e.g., the base station. Each SN $s_i$ requires a set of RNs, denoted by $\mathcal{R}_i$, for cooperative communication. We call $\mathcal{R}_i$ $s_i$'s RN *bundle*. Assume that for each $s_i$, there exists at least one SN $s_j$, such that $\mathcal{R}_i \cap \mathcal{R}_j \neq \emptyset$. This assumption necessitates the participation of $s_i$ in the auction because of the competition, and is called *competition assumption*.

For the transmission model, we assume that when node $u$ transmits a signal to node $v$ with power $P_u$, the signal-to-noise ratio (SNR) at node $v$, denoted by $SNR_{uv}$, is $SNR_{uv} = \frac{P_u}{N_0 \cdot ||u,v||^\gamma}$, where $N_0$ is the abient noise, $||u,v||$ is the Euclidean distance between nodes $u$ and $v$, and $\gamma$ is the path loss exponent which is between 2 and 4 in general, depending on the characteristics of the communication medium. To mitigate the interference, we assume that there are enough orthogonal channels available.

### B. Multi-Relay Cooperative Communication

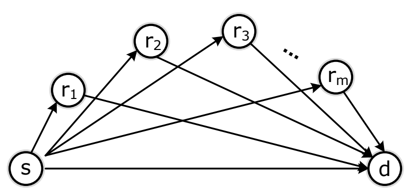



Fig. 2. Multi-relay cooperative communication

We consider the *multi-relay cooperative communication* as shown in Fig. 2. We adopt the communication model proposed in [12]. The whole transmission completes in two time slots. In the first time slot, the SN $s$ transmits data to the destination node $d$. Meanwhile, due to the broadcast nature of the wireless transmission, the RNs can overhear the transmitted signal. In the second time slot, the best RN selected by the destination node forwards the received data to the destination node after processing according to the underlying cooperative communication mode. For example, if the amplify-and-forward mode is used, the achievable data capacity from $s$ to $d$ using $\mathcal{R}$ is

$$\begin{aligned} &C_{AF}(s, \mathcal{R}_i, d) \\ =&\frac{W}{2}\log_2\left(1 + SNR_{sd} + \max_{r_i \in \mathcal{R}_i} \frac{SNR_{sr_i} SNR_{r_i d}}{SNR_{sr_i} + SNR_{r_i d} + 1}\right), \end{aligned}$$

where $W$ is the bandwidth of the communication channel. Note that auction mechanisms designed in this paper are independent of the cooperative communication mode. It is also worth mentioning that how to select the optimal RN set for each SN is out of the scope of this paper.

### C. Auction Formulation

In our auction, the administrator is the seller and the SNs are the buyers (also called bidders) who bid for RNs. As is in economics, we assume that bidders' private valuations on their RN bundles, denoted by $v_1, v_2, \ldots, v_n$, are drawn independently from known prior distribution functions. This assumption is called *Bayesian setting*. These distribution functions can be obtained based on the transaction history. We denote by $F_i$ the distribution function from which $v_i$ is drawn, and by $f_i(z) = \frac{d}{dz}F_i(z)$ the corresponding density function. Each bidder is asked to declare its desired RN bundle $\mathcal{R}_i$ and its valuation $w_i$ on $\mathcal{R}_i$. Note that $w_i$ can be different from $v_i$. We use $b_i = (w_i, \mathcal{R}_i)$ to denote bidder $s_i$'s bid.

An auction mechanism takes as input the bid vector consisting of all bidders' bids, denoted by $\mathbf{b} = (b_1, b_2, \ldots, b_n)$. We use $\mathbf{b}_{-i}$ to denote the bids of bidders except $s_i$ and then have $\mathbf{b} = (b_i, \mathbf{b}_{-i})$. The auction mechanism then determines the winner vector $\mathbf{x}(\mathbf{b}) = (x_1, x_2, \ldots, x_n)$ and the charging prices $\mathbf{p}(\mathbf{b}) = (p_1, p_2, \ldots, p_n)$, where $x_i = 1$ if bidder $s_i$ wins and is allocated the RN bundle $\mathcal{R}_i$ while $x_i = 0$ otherwise, and $p_i$ is the price bidder $s_i$ needs to pay the seller. It is clear that $p_i = 0$ if $x_i = 0$. The allocation of RNs must satisfy the condition: $\mathcal{R}_i \cap \mathcal{R}_j = \emptyset$, for all $x_i = x_j = 1$. We assume that the bidders are *single-parameter*, i.e., the valuation is $v_i$ if bidder $i$ wins the auction and 0 otherwise. We further assume that the utility of bidder $i$ is a quasi-linear function, i.e., $u_i(\mathbf{b}) = v_i x_i - p_i$.

The *revenue* of an auction mechanism is the sum of the payment $\sum_{s_i \in \mathcal{S}} p_i$ from all bidders.

**Definition 1:** An auction mechanism is *truthful* (in expectation) if and only if for every $s_i$ and $\mathbf{b}_{-i}$, bidder $s_i$'s expected utility for bidding its true valuation $v_i$, is at least its expected utility for bidding any other value, i.e., $u_i(b'_i, \mathbf{b}_{-i}) \geq u_i(b_i, \mathbf{b}_{-i})$, where $b'_i = (v_i, \mathcal{R}_i)$ and $b_i = (w_i, \mathcal{R}_i)$. □

The auction mechanisms designed in this paper are based on the Myerson mechanism. The Myerson mechanism is based on the characterization of truthful mechanism [7, Theorem 13.6].

**Theorem 1:** Under Bayesian setting, an auction mechanism is truthful if and only if, for any bidder $i$ and any fixed choice of bids $b_{-i}$ by other bidders,

1) $x_i(b_i)$ is monotonically nondecreasing, i.e., if bidder $i$ wins by bidding $w_i$, then it also wins by bidding $w_i' \geq w_i$, and
2) the payment $p_i$ for any winning bidder $i$ is set to the *critical value* $t_i$, which is the value such that bidder $i$ wins if $w_i > t_i$ and loses otherwise.

In Myerson mechanism, the concept of *virtual valuation* is introduced.

**Definition 2:** The *virtual valuation* of bidder $i$ with valuation $v_i$ is

$$\psi_i(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}, \tag{1}$$

where $f_i(z)/(1 - F_i(z))$ is called the *hazard rate* of a distribution and assumed to be monotonically nondecreasing.□

Given valuations, $v_i$, and corresponding virtual valuations, $\psi_i(v_i)$, the *virtual surplus* of allocation $\mathbf{x}$ is $\sum_{s_i \in \mathcal{S}} \psi_i(v_i)x_i$. It is proved that any truthful auction mechanism has expected revenue equal to its expected virtual surplus [7, Theorem 13.10].

**Theorem 2:** The expected profit of any truthful mechanism under the Bayesian setting is equal to its expected virtual surplus.

## IV. Auction with Optimal Revenue

Based on Theorems 1 and 2, the designed auction mechanisms need to maximize the virtual surplus while satisfying the two properties in Theorem 1. The well-known VCG-based auction mechanism fits in this category. To apply this mechanism, it is required to develop an optimal RN allocation algorithm with the objective of revenue maximization.

### A. VCG-based Auction Design

The RN allocation problem can be defined as follows:

**Definition 3:** Given a bid vector $\mathbf{b}$ and a set $\mathcal{R}$ of RNs, the RN allocation problem is to find a winner vector $\mathbf{x}$, such that $\sum_{s_i \in \mathcal{S}} \psi_i(v_i)x_i$ is maximized under the constraint that $\mathcal{R}_i \cap \mathcal{R}_j = \emptyset$ whenever $x_i = x_j = 1$. □

We can transform the RN allocation problem into the Maximum Weighted Independent Set (MWIS) problem as follows: For any instance of the RN allocation problem, we can construct a graph $G = (V, E)$. For each SN $s_i$, we have a vertex $s_i$ in $V$. Each vertex $i$ is associated with weight $\psi(w_i)$. There is an edge $(s_i, s_j) \in E$ between $s_i$ and $s_j$ if and only if $\mathcal{R}_i \cap \mathcal{R}_j \neq \emptyset$. Let $N(s_i) = \{s_j | (s_i, s_j) \in E\}$ denote the set of neighbors of $s_i$. Let $d(s_i)$ denote the degree of node $s_i$ in $G$. It is straightforward to see that solving the RN allocation problem is equivalent to finding an MWIS in $G$. The MWIS problem can be solved using *integer linear programming* (ILP) as shown below:

$$\textbf{ILP}: \quad \max \sum_{s_i \in \mathcal{S}} \psi_i(w_i)x_i \tag{2}$$

$$s.t. \quad x_i + x_j \leq 1, \quad (s_i, s_j) \in E \tag{3}$$

$$x_i \in \{0, 1\}, \quad s_i \in \mathcal{S}. \tag{4}$$

We denote the optimal value of the above ILP by $\Psi(\mathcal{S})$ and the optimal solution by $\mathbf{x}^*$. The detailed design of our VCG-based auction is as follows:

1) Given the bid vector $\mathbf{b}$ and $\{F_i\}$, compute virtual valuations $\psi(w_i)$ by Eq. (1).
2) Use **ILP** ((2)-(4)) to compute an optimal allocation $\mathbf{x}^*$.
3) Set $p_i = \psi_i^{-1}(\Psi(\mathcal{S} \setminus \{s_i\}) - (\Psi(\mathcal{S}) - x_i^*\psi_i(w_i)))$ if $x_i = 1$, and set $p_i = 0$ otherwise, where $\psi_i^{-1}$ is the inverse of $\psi_i$ and $\Psi(\mathcal{S} \setminus \{s_i\})$ is the optimal value of the MWIS problem with $s_i$ removed.

### B. Proof of Truthfulness

To prove the truthfulness of the VCG-based auction mechanism, we show that the winner determination is monotonically nondecreasing and the price each winner needs to pay is a critical value.

**Lemma 1:** The winner determination in the VCG-based auction mechanism is monotonically nondecreasing. □

*Proof:* Assume that bidder $s_i$ wins the auction by bidding $w_i$. Let $\mathbf{x}$ be the winner vector. Assume that bidder $s_i$ bids $w_i' \geq w_i$. Due to the monotonicity assumption on the hazard rate of the distribution function, we know that $\psi_i(w_i') \geq \psi_i(w_i)$. Hence having the same winner vector $\mathbf{x}$ can maximize the revenue under the new bid vector as well.■

**Lemma 2:** The price $p_i$ is a critical value for each winner bidder $s_i$ in the VCG-based auction mechanism. □

*Proof:* In the VCG-based auction mechanism, the payment of each winner bidder is calculated based on the opportunity cost that its presence introduces to all the other bidders. Therefore, if the bidder bids less than this price, it will not be selected as the winner, since the revenue is higher. ■

**Theorem 3:** The VCG-based auction is truthful. □

*Proof:* Lemmas 1 and 2 together prove the theorem, based on Theorem 1. ■

### C. Time Complexity

Although the VCG-based auction offers the revenue maximization while guaranteeing truthfulness, it has heavy computing complexity, due to the requirement for solving the MWIS problem. It has been proved that the MWIS problem is an NP-hard problem [1]. To make the matter worse, it is known that no polynomial time approximation algorithm for the Maximum Independent Set (MIS) problem (MWIS with unit weight on the vertex) can provide an $O(n^{1-\varepsilon})$ guarantee for any $\varepsilon > 0$, unless $P = NP$ [2]. Therefore, we need to trade revenue for computing efficiency.

## V. Auction with Suboptimal Revenue

In this section, we design a truthful auction mechanism with low time complexity, called *suboptimal auction mechanism*.

### A. Suboptimal Auction Design

To overcome the high computing complexity of the VCG-based auction mechanism, one may propose to replace the optimal algorithm with an approximation algorithm to reduce the running time. However, on one hand, it has been shown that a VCG-based auction mechanism with an approximation algorithm does not preserve the truthfulness [7]. On the other hand, it is unlikely to have an approximation algorithm with approximation ratio better than $O(n^{1-\varepsilon})$. We thus focus our attention on designing a non-VCG auction.

Our auction mechanism consists of two phases: *winner determination* and *pricing*.

In the winner determination phase, as illustrated in Algorithm 1, the conflict graph based on the submitted bids is first constructed. Next, the virtual valuation $\psi_i(w_i)$ for each bidder is calculated. To have a criterion to sort the bidders, $\alpha_i = \frac{\psi_i(w_i)}{d(s_i)}$ is computed for each bidder. Note that other criteria can also be adopted as long as $\alpha_i$ is a monotonic function of $w_i$. The reason that we use $\frac{\psi_i(w_i)}{d(s_i)}$ is that $d(s_i)$ directly affects the number of bidders disqualified from the auction. The bidders are then sorted in a nondecreasing order in terms of $\alpha_i$. Next, the winners are determined in a greedy manner. In each iteration, the first bidder $s_i$ in the list is selected as a winner. All the neighbors of $s_i$ in the conflict graph are then eliminated from the auction. This process repeats until the list becomes empty.

**Algorithm 1**: Winner Determination Phase

**input** : The bid vector $\mathbf{b}$
**output**: The winner vector $\mathbf{x}$
1 $x_i \leftarrow 0$, for all $s_i \in \mathcal{S}$;
2 Construct the conflict graph, $G = (V, E)$;
3 $\psi_i(w_i) \leftarrow w_i - \frac{1-F_i(w_i)}{f_i(w_i)}$, for all $s_i \in \mathcal{S}$;
4 $\alpha_i \leftarrow \frac{\psi_i(w_i)}{d(\mathcal{R}_i)}$, for all $s_i \in \mathcal{S}$;
5 Sort the bidders in a nonincreasing order according to $\alpha_i$. Let $L$ denote the sorted list;
6 **while** $L \neq \emptyset$ **do**
7     Let $s_i$ be the next bidder in $L$;
8     $x_i \leftarrow 1$;
9     $L \leftarrow L \setminus \{s_i\}$;
10     **forall** $s_j \in N(s_i)$ **do** $L \leftarrow L \setminus \{s_j\}$;
11 **return** $\mathbf{x}$;

In the pricing phase, as illustrated in Algorithm 2, the payment that each winner bidder needs to pay is determined. The basic idea is to determine the critical value for each bidder. In order to find the critical value, we temporarily remove $s_i$ from $\mathcal{S}$ and find the first bidder whose selection disqualifies $s_i$ from winning the auction. Such bidder is guaranteed to exist, because of the competition assumption in Section III.

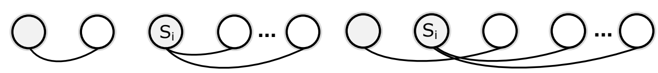
### *B. Proof of Truthfulness*

**Lemma 3:** The winner determination in the suboptimal auction mechanism is monotonically nondecreasing. □

*Proof:* Assume that bidder $s_i$ wins the auction by bidding $w_i$. We prove that it can also win by bidding $w_i' \geq w_i$. Due to the nondecreasing monotonicity of $\psi_i(w_i)$, we have $\psi_i(w_i') \geq \psi_i(w_i)$. Let $L$ and $L'$ denote the sorted list when $s_i$ bids $w_i$ and $w_i'$, respectively. Let $q$ and $q'$ denote the position of $s_i$ in $L$ and $L'$, respectively. Since $\alpha_i' = \frac{\psi_i(w_i')}{d(s_i)} \geq \alpha_i = \frac{\psi_i(w_i)}{d(s_i)}$, we have $q' \leq q$, as shown in Fig. 3. It is clear that all the bidders who are behind $s_i$ in $L$ are still behind $s_i$ in $L'$. Therefore $s_i$ will still selected by Algorithm 1 as a winner. ■

**Algorithm 2**: Pricing Phase

**input** : The bid vector $\mathbf{b}$ and the winner vector $\mathbf{x}$
**output**: The pricing vector $\mathbf{p}$
1 Construct the conflict graph $G = (V, E)$;
2 $\psi_i(w_i) \leftarrow w_i - \frac{1-F_i(w_i)}{f_i(w_i)}$, for all $s_i \in \mathcal{S}$;
3 $\alpha_i \leftarrow \frac{\psi_i(w_i)}{d(\mathcal{R}_i)}$, for all $s_i \in \mathcal{S}$;
4 Sort the bidders in a nonincreasing order according to $\alpha_i$. Let $L$ denote the sorted list;
5 **forall** $s_i \in \mathcal{S}$ **do**
6     **if** $x_i = 1$ **then**
7         $L' \leftarrow L \setminus \{s_i\}$;
8         **while** $L' \neq \emptyset$ **do**
9             Let $s_j$ be the next bidder in $L'$;
10             $L' \leftarrow L' \setminus \{s_j\}$;
11             **if** $s_i \in N(s_j)$ **then**
12                 $p_i \leftarrow \psi_i^{-1}(\alpha_j d(\mathcal{R}_i))$; **break**;
13             **forall** $s_k \in N(s_j)$ **do** $L' \leftarrow L' \setminus \{s_k\}$;
14     **else** $p_i \leftarrow 0$;
15 **return** $\mathbf{p}$;

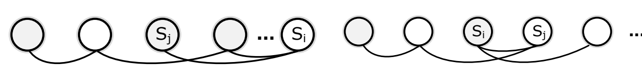


(a) $s_i$ bids $w_i$ (b) $s_i$ bids $w_i' \geq w_i$

Fig. 3. Monotonicity of the winner determination in the suboptimal auction mechanism

**Lemma 4:** The price $p_i$ is a critical value for each winner bidder $s_i$ in the suboptimal auction mechanism. □

*Proof:* Let $s_j$ be the first bidder in the list, whose selection can disqualify $s_i$. Let $t_i = \psi_i^{-1}(\alpha_j d(s_i))$. If bidder $s_i$ bids $w_i < t_i$, then $\alpha_i = \frac{\psi_i(w_i)}{d(s_i)} < \frac{\psi_i(t_i)}{d(s_i)} = \alpha_j$. As shown in Fig. 4(a), we know that $s_i$ will be behind $s_j$ in the list and thus will be eliminated from the auction. If bidder $s_i$ bids $w_i > t_i$, $s_i$ will be ahead of any bidders whose selection can disqualify $s_i$, since $s_j$ is the first of such bidders, as shown in Fig. 4(b). Therefore $s_i$ will be selected as a winner. Obviously, $t_i$ is the critical value of $s_i$. ■



(a) $w_i < t_i$ (b) $w_i > t_i$

Fig. 4. Illustration of the critical value searching

**Theorem 4:** The suboptimal auction mechanism given in Algorithms 1 and 2 is truthful. □

*Proof:* Lemmas 3 and 4 together prove the theorem, based on Theorem 1. ■

### *C. Time Complexity*

**Theorem 5:** The running time of the suboptimal auction mechanism is $O(n^2(m+n))$. □

*Proof:* In Algorithm 1, the construction of the conflict graph (Lines 1–4) dominate the running time and takes $O(n^2 m)$ time. In Algorithm 2, it takes $O(n^2 m)$ time to finish Lines 1–4. The for-loop (Lines 5 to 5) takes $O(n^3)$ time, because the while-loop takes $O(n^2)$ time and the for-loop runs $n$ iterations. Therefore the total running time the suboptimal auction mechanism is $O(n^2(m+n))$. ■

## VI. Evaluations

In this section, we evaluate the performance of the designed auction mechanisms through experiments.

### A. Experiment Setup

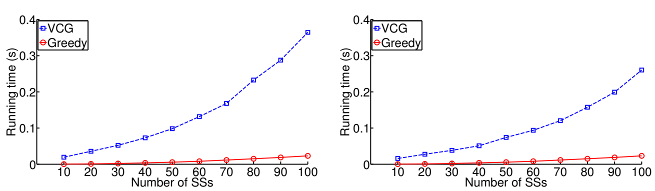
Throughout all the experiments, we assumed that the destination node is located at $(0, 0)$. Both the SNs and the RNs were randomly distributed in a $1000m \times 1000m$ square. The number of SNs varied from 10 to 100, with the increment of 10. For the valuation distribution function, we considered the uniform distribution $F_i(z) = z$ in the range of $(0, 1]$, and the exponential distribution $F_i(z) = 1 - e^{-z}$. These two functions were also used in [5]. For each setting, we randomly generated 100 instances and averaged the results. We use $VCG$ to denote the VCG-based optimal auction mechanism and $Greedy$ to denote the suboptimal auction mechanism.

*1) RN Bundle Selection:* As to the RN bundle selection for each SN $s_i$, we assume that $s_i$ first chooses all the RNs, via which it can have higher capacity than direct transmission. The SN $s_i$ then randomly select $m_i$ RNs as its RN bundle, where $m_i$ is a random number uniformly distributed over $[1, 10]$. It is worth mentioning that the auction mechanisms designed in this paper is independent of the RN bundle selection algorithm.

*2) Performance Metric:* The performance metrics considered in paper include the running time and the revenue.

### B. Result Analyses

*1) Running Time:* Fig. 5 shows the running time of the designed auction mechanisms. In particular, Fig. 5(a) and Fig. 5(b) show the running time using the uniform distribution and the exponential distribution, respectively. In both figures, we observe that the running time of VCG grows exponentially with the number of SNs $(n)$. Whereas the running time of $Greedy$ grows very slowly. When $n = 100$, the running time of $VCG$ is 15 times more than that of $Greedy$ for the uniform distribution and 11 times for the exponential distribution.

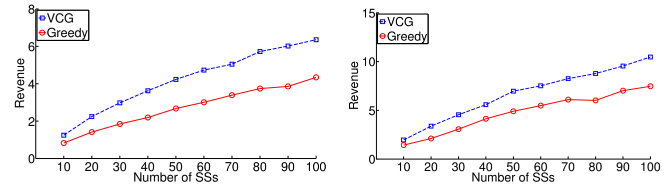



(a) Uniform distribution (b) Exponential distribution

Fig. 5. Running time

*2) Revenue:* We now show the results of the experiments on the revenue. As shown in Fig. 6, the revenue of $Greedy$ is lower than that of $VCG$ as expected. However, considering the significant improvement of the running time, the revenue degradation of $Greedy$ compared to $VCG$ is acceptable.

To quantify the revenue degradation of $Greedy$, we illustrate the degradation ratio in Fig. 7. Our observation is that, in most cases, $Greedy$ has lower degradation ratio using the exponential distribution than it does using the uniform distribution. The reason is that the valuation is distributed more even in the uniform distribution than it is in the exponential distribution. Hence, when the exponential distribution is used,




(a) Uniform distribution (b) Exponential distribution

Fig. 6. Revenue

as long as $Greedy$ selects the winners with significantly large virtual valuations, the resulting revenue would not be significantly less than that of $VCG$.

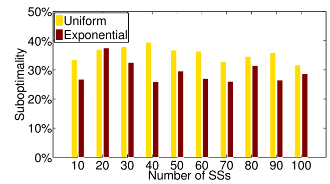



Fig. 7. Comparison between distribution functions

## VII. Conclusion

In this paper, we considered how to allocate the relay nodes and price the source nodes using auction mechanisms, such that the revenue is maximized. We formulated the problem as a combinatorial auction. We first proposed a VCG-based auction mechanism, which maximizes the revenue while enforcing the truthfulness. The high time complexity of the VCG-based auction mechanism motivated us to design another truthful auction mechanism, which significantly reduces the time complexity without severely sacrificing the revenue.

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